SWITCHING CIRCUIT DESIGNS from BIOLOGICAL EVOLUTION

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Mathematics Department
SUNY, New Paltz

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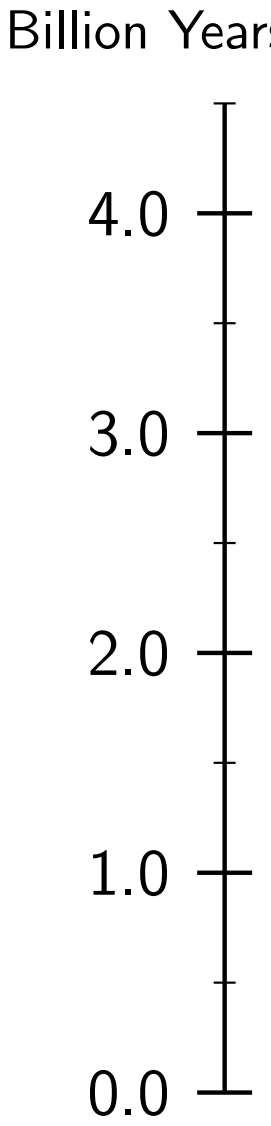
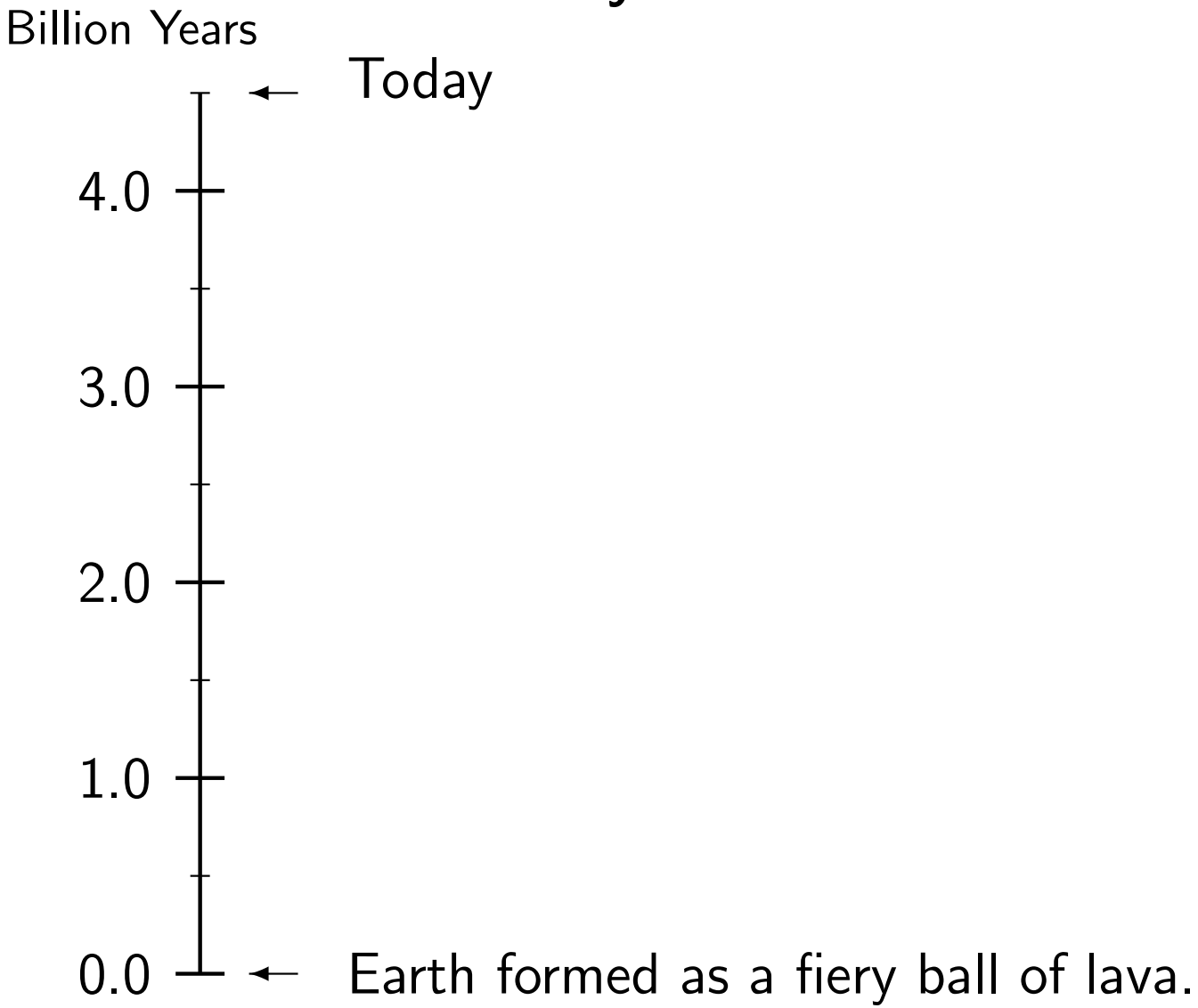
1. Biological Evolution

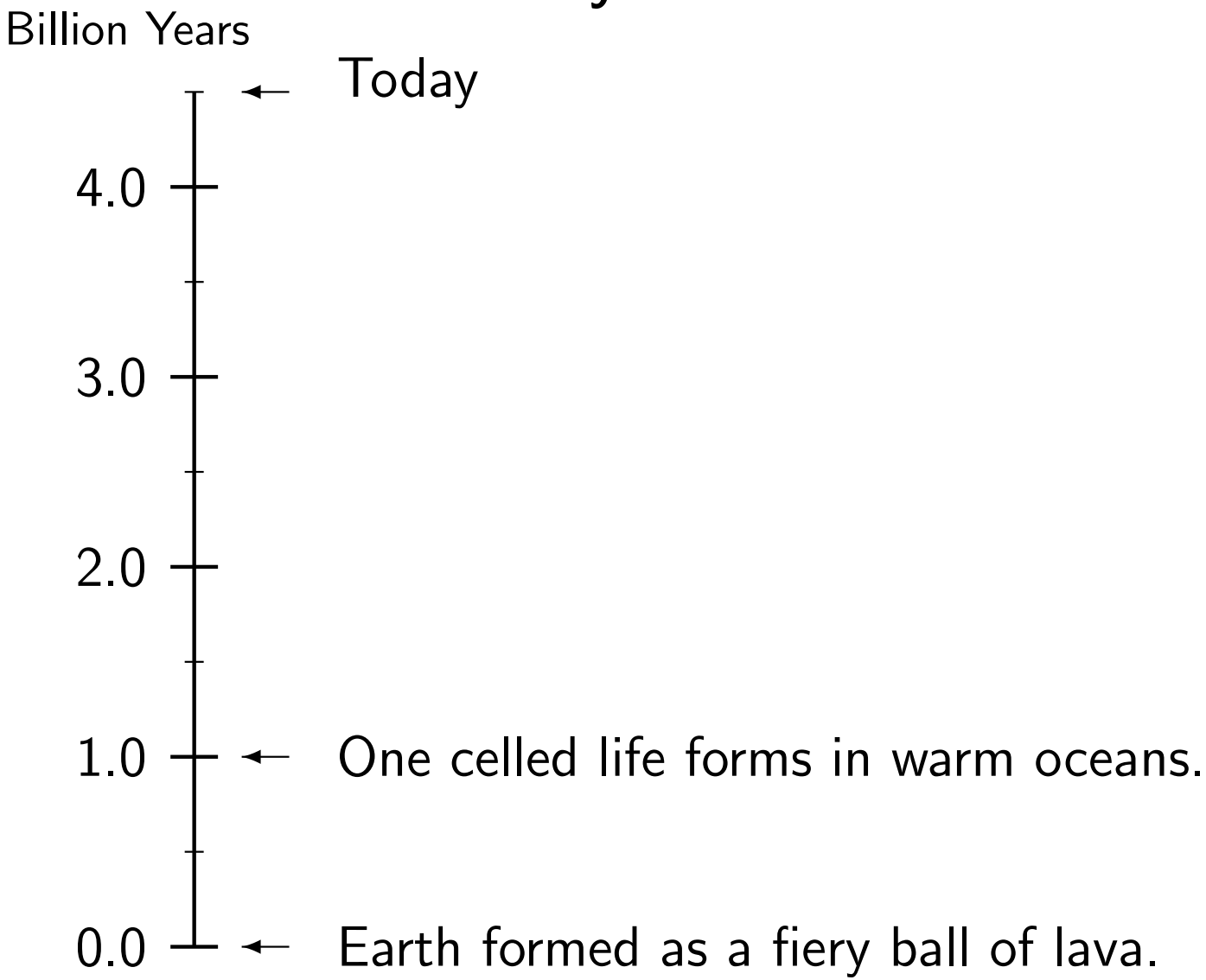
History of Life on Earth



History of Life on Earth



History of Life on Earth



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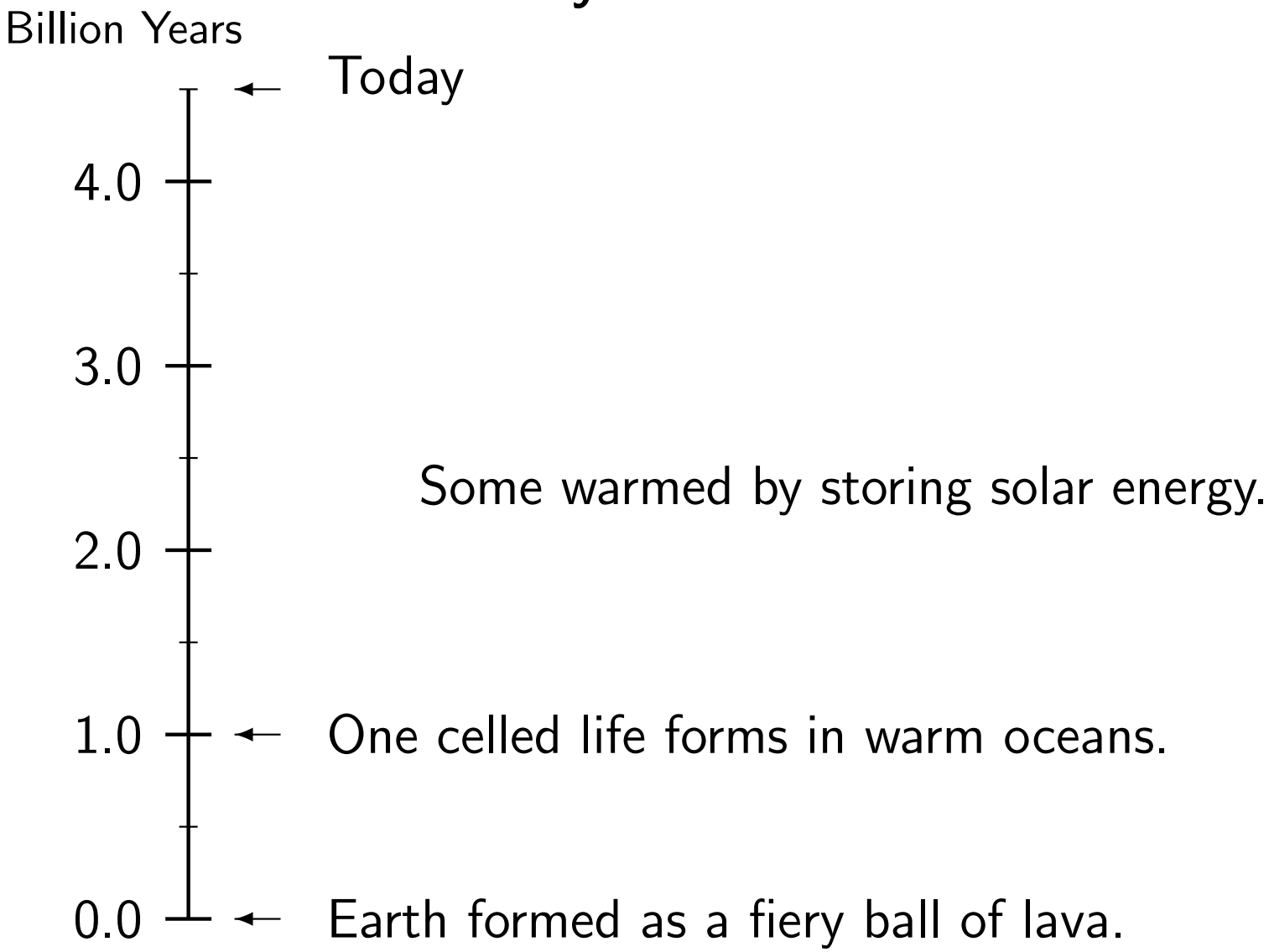
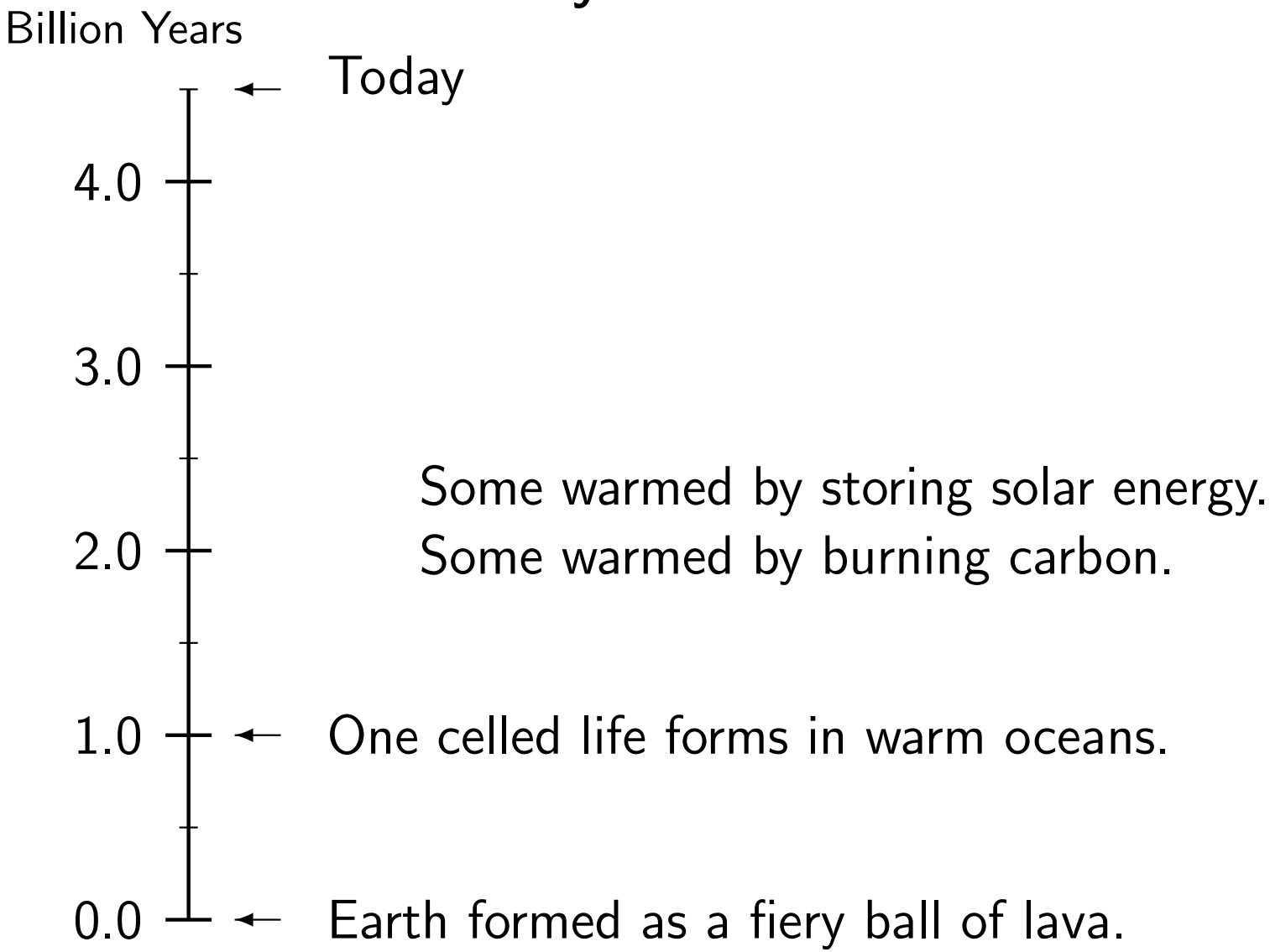


History of Life on Earth



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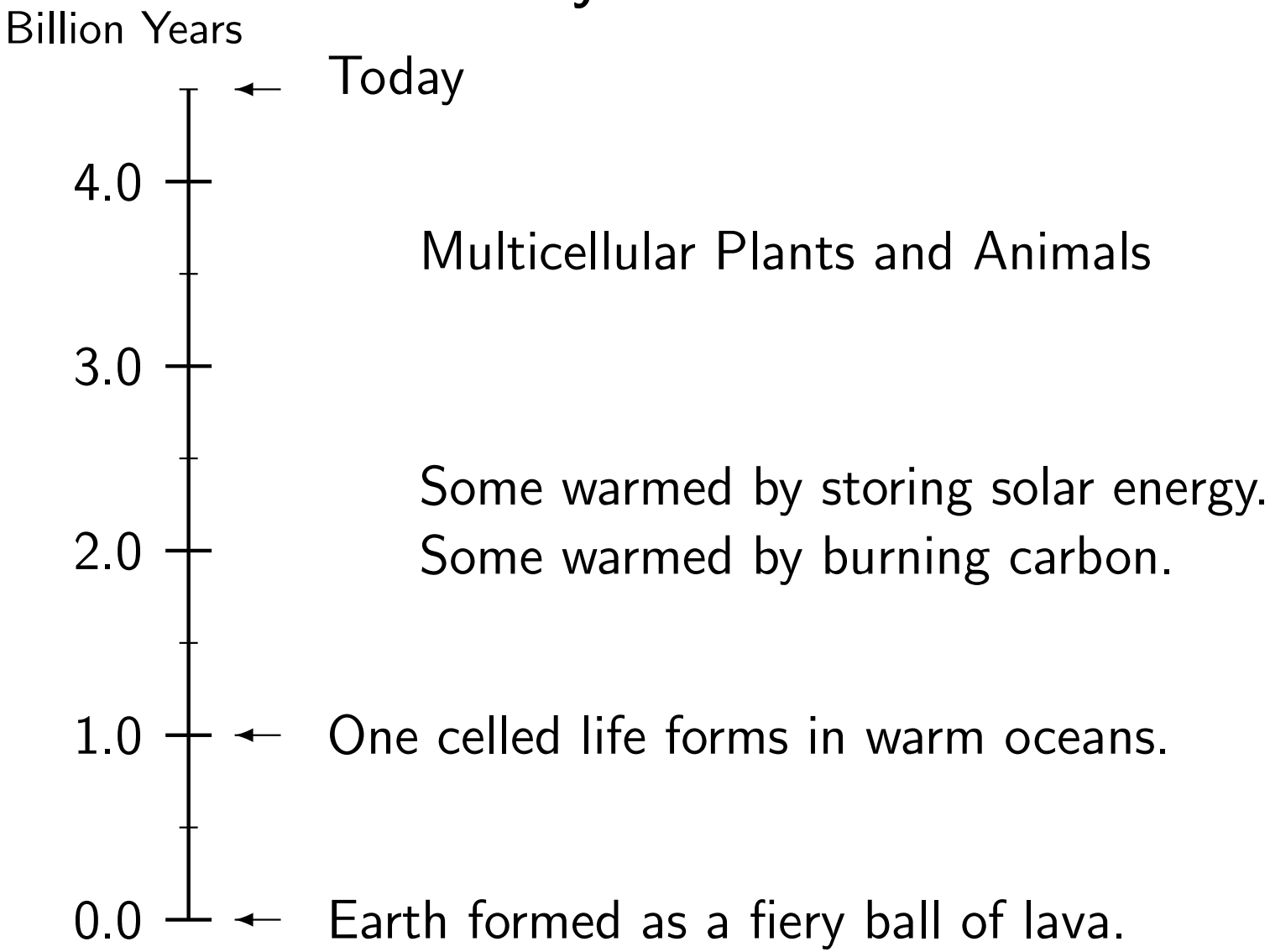
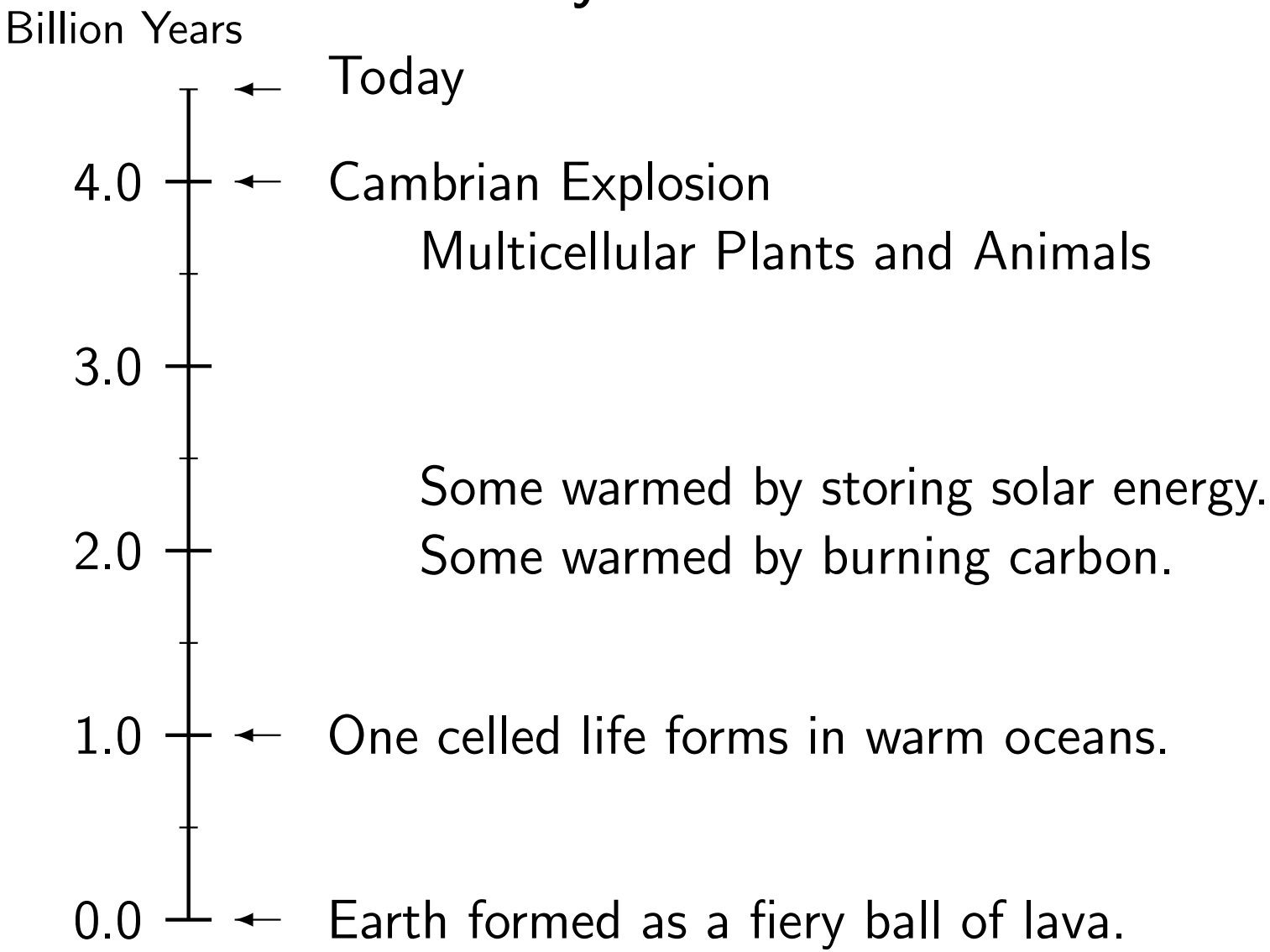
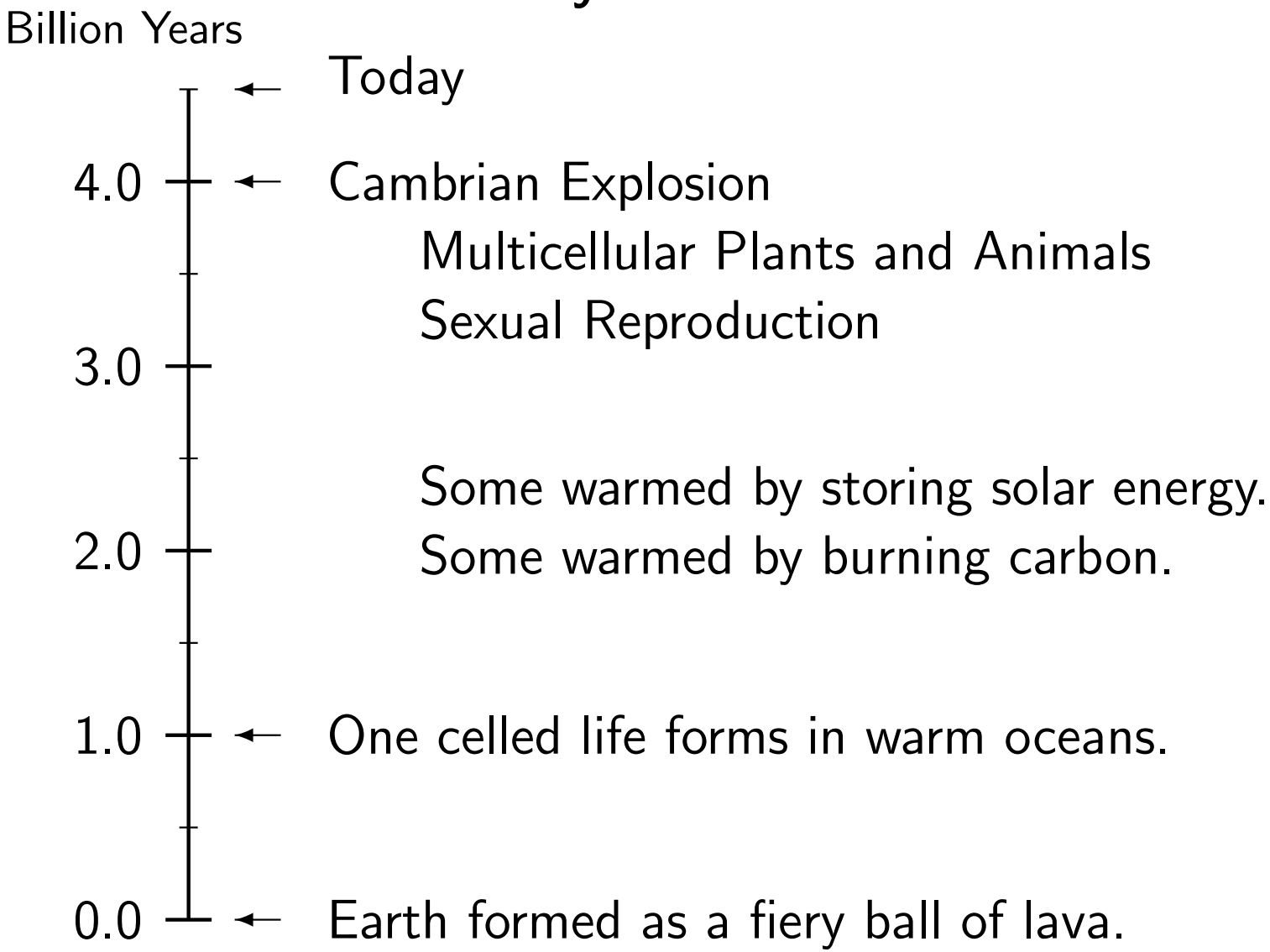


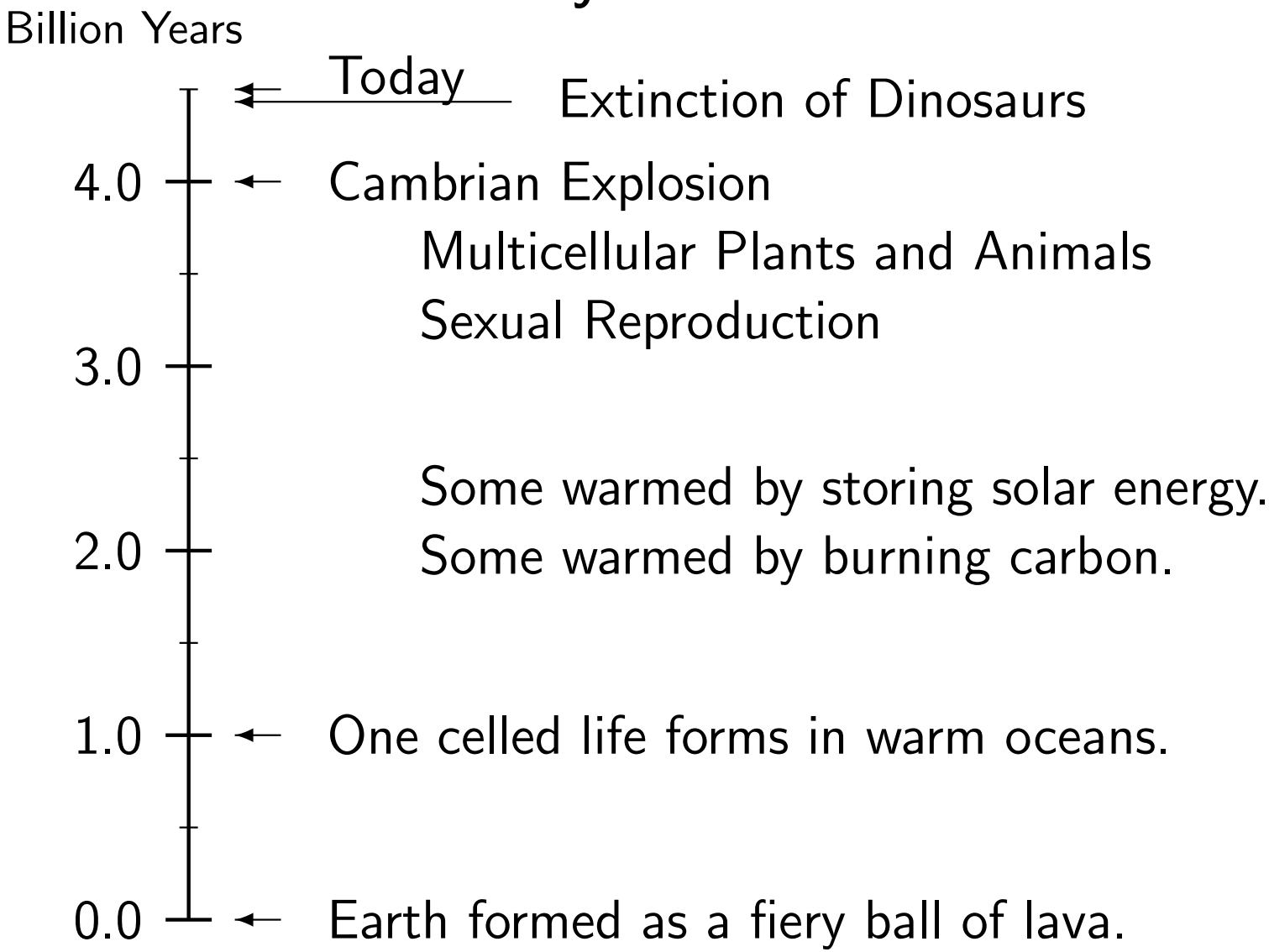
History of Life on Earth



History of Life on Earth



History of Life on Earth



Why is biological evolution so slow?

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Genotype: a design for an individual.

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Phenotype: individual built from that design.

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Evolutionary process:

.. genotypes $\overset{build}{\Rightarrow}$ phenotypes $\overset{select}{\Rightarrow}$ genotypes $\overset{build}{\Rightarrow}$ phenotypes ..

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Humans:

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Humans: 30 years

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Humans:	30 years	3 days

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Humans:	30 years	3 days	30 years

Why is biological evolution so slow?

Genotype: a design for an individual.

Phenotype: individual built from that design.

Evolutionary process:

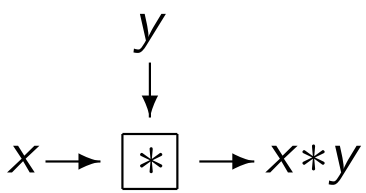
.. genotypes $\overset{build}{\Rightarrow}$ phenotypes $\overset{select}{\Rightarrow}$ genotypes $\overset{build}{\Rightarrow}$ phenotypes ..

Humans:	30 years	3 days	30 years

Conclusion: Biological evolution is slow because it uses phenotypes to test the fitness of genotypes.

2. Switching Circuits

A **binary gate** is a small electronic chip with a set, say $\{0, 1, 2\}$, of input/output values. Given inputs values x and y it produces an output denoted by $x*y$



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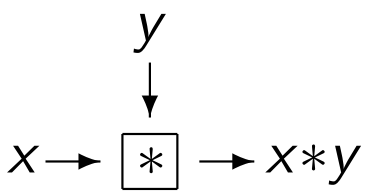

	$*$	0	1	2
	0	2	1	2
$x \rightarrow$	1	1	0	0
	2	0	0	1

$y \downarrow$

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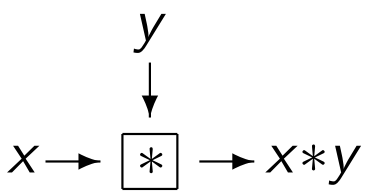
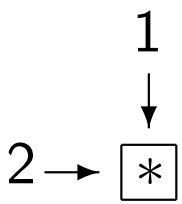


	$*$	0	1	2
	0	2	1	2
$x \rightarrow$	1	1	0	0
	2	0	0	1

For example,



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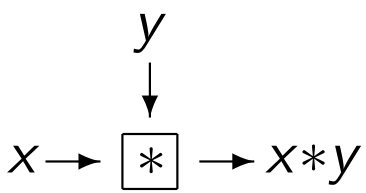

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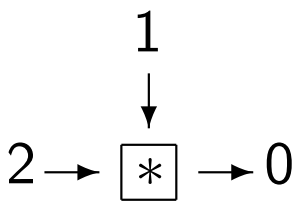
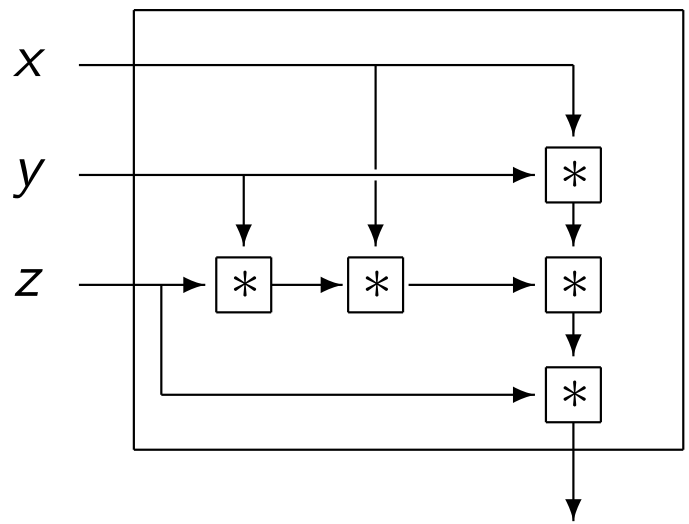


$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Switching Circuit



$*$	0	1	2
0	2	1	2
1	1	0	0
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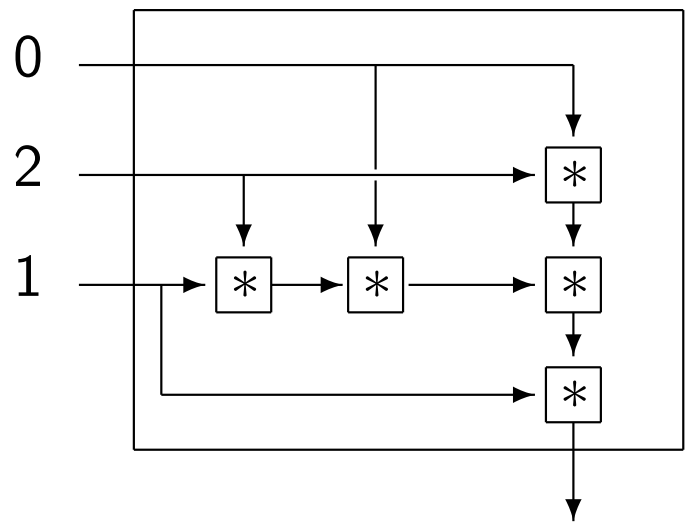
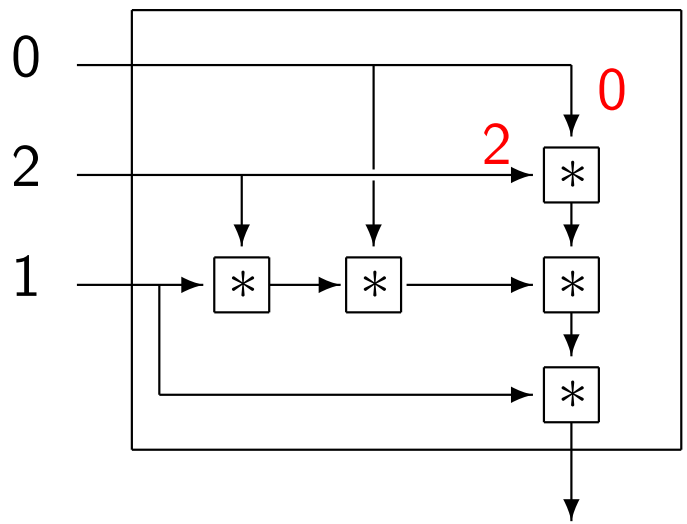
Switching Circuit



$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Switching Circuit



$*$	0	1	2
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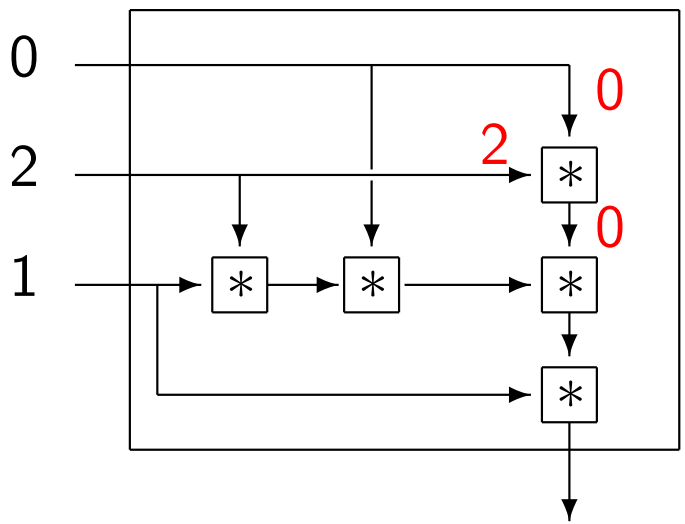
1	1	0	0
2	0	0	1

Switching Circuit



$*$	0	1	2
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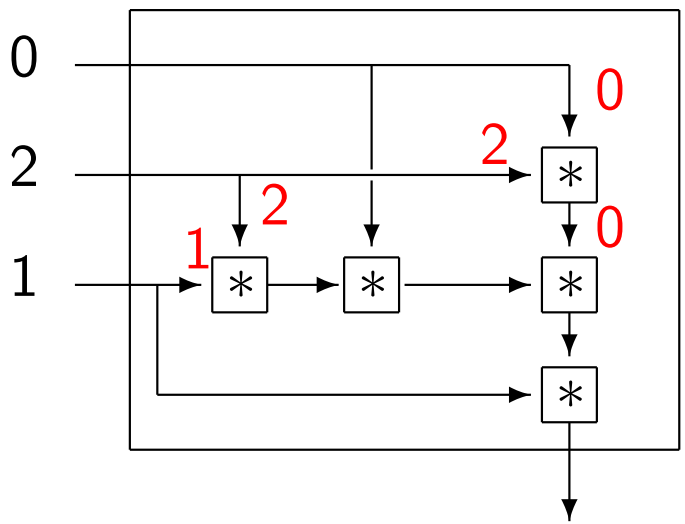
2	0	0	1

Switching Circuit



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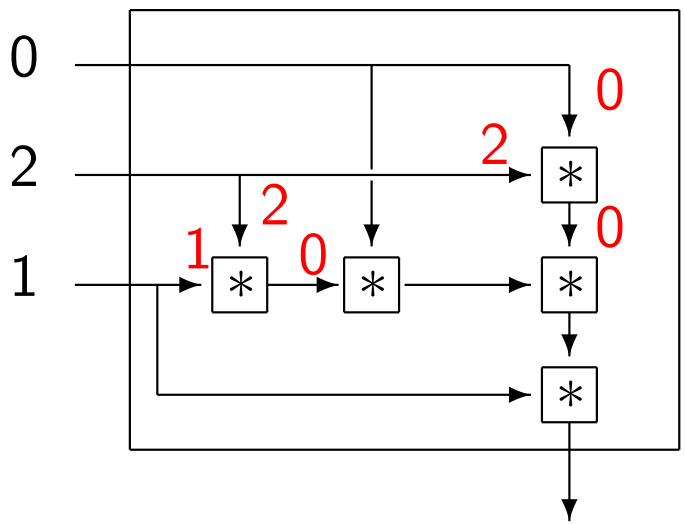
1	1	0	0
2	0	0	1

Switching Circuit



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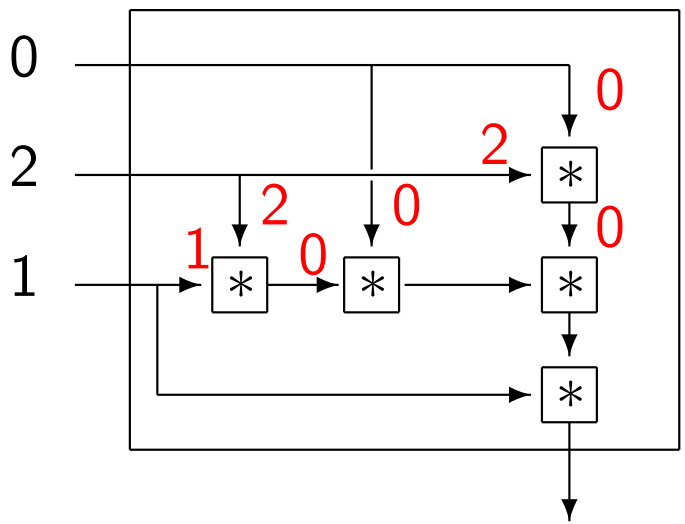

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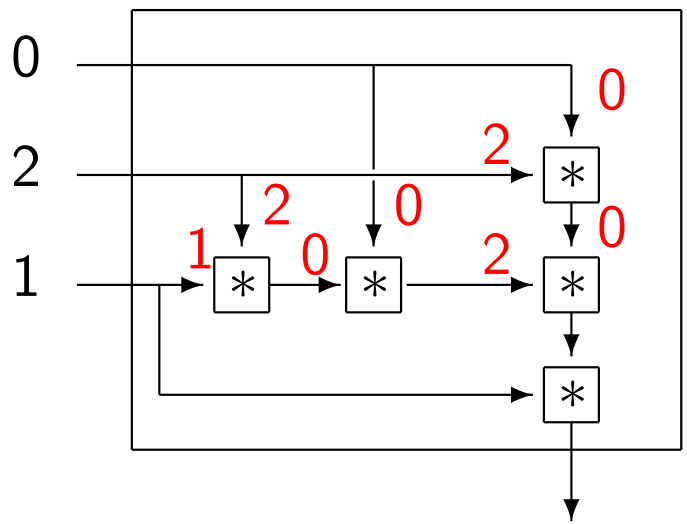
2	0	0	1

Switching Circuit



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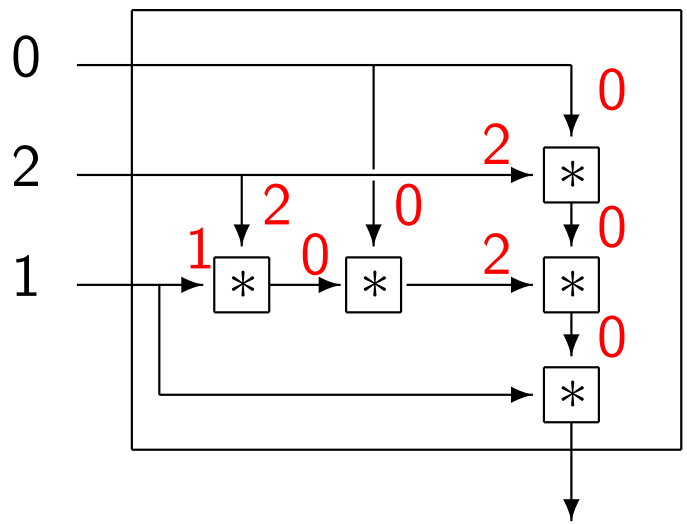
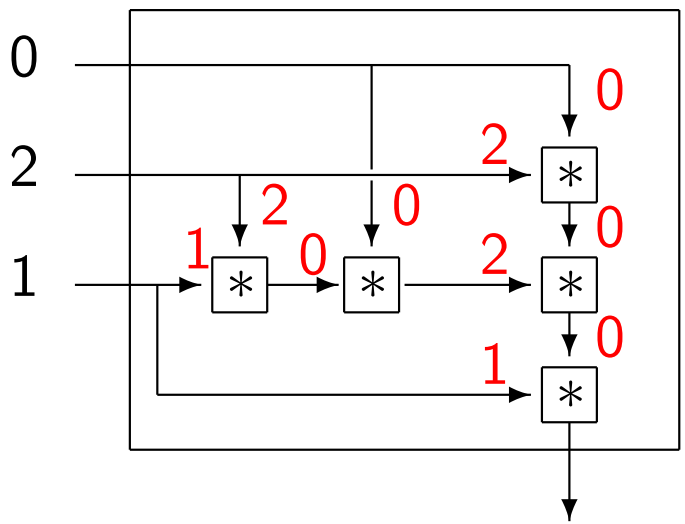
Switching Circuit



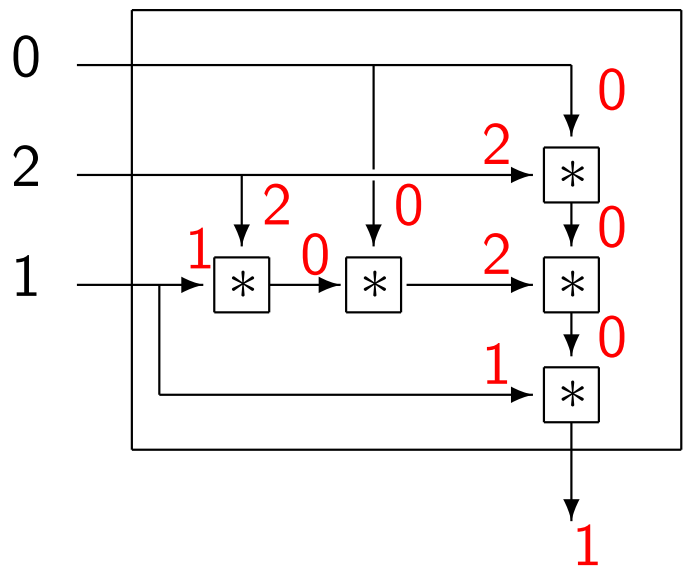
$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Switching Circuit



$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

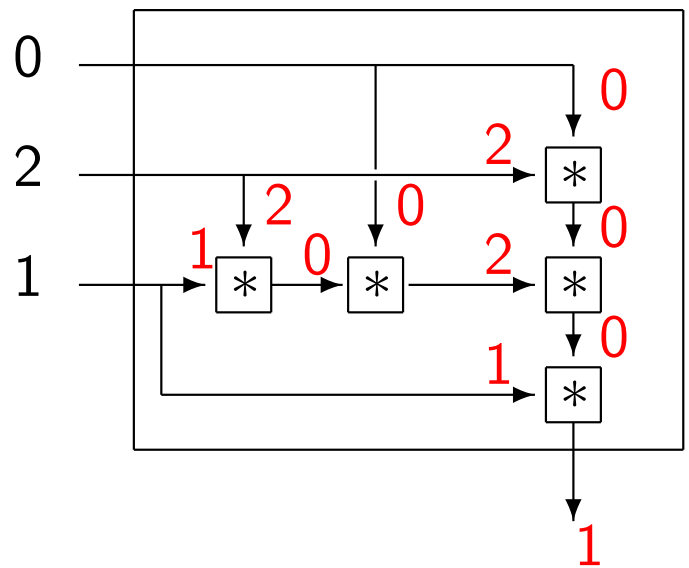
Switching Circuit



x	y	z	Output
0	0	0	
0	0	1	
0	0	2	
0	1	0	
0	1	1	
0	1	2	
0	2	0	
0	2	1	1
0	2	2	
1	0	0	
1	0	1	
:	:	:	
2	1	2	
2	2	0	
2	2	1	
2	2	2	

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

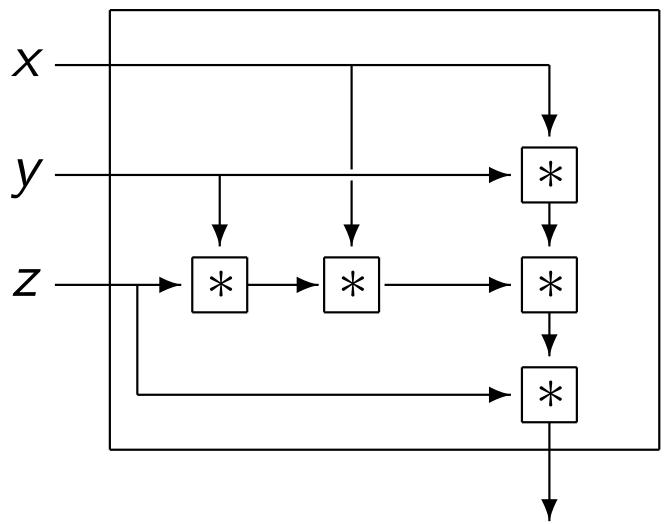
Switching Circuit



x	y	z	Output
0	0	0	2
0	0	1	1
0	0	2	0
0	1	0	2
0	1	1	1
0	1	2	0
0	2	0	2
0	2	1	1
0	2	2	0
1	0	0	1
.	.	.	.
.	.	.	.
2	1	2	0
2	2	0	2
2	2	1	1
2	2	2	0

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

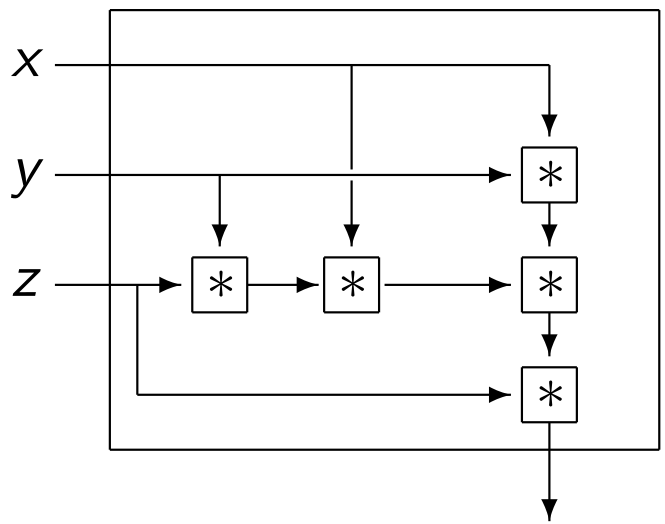
Switching Circuit

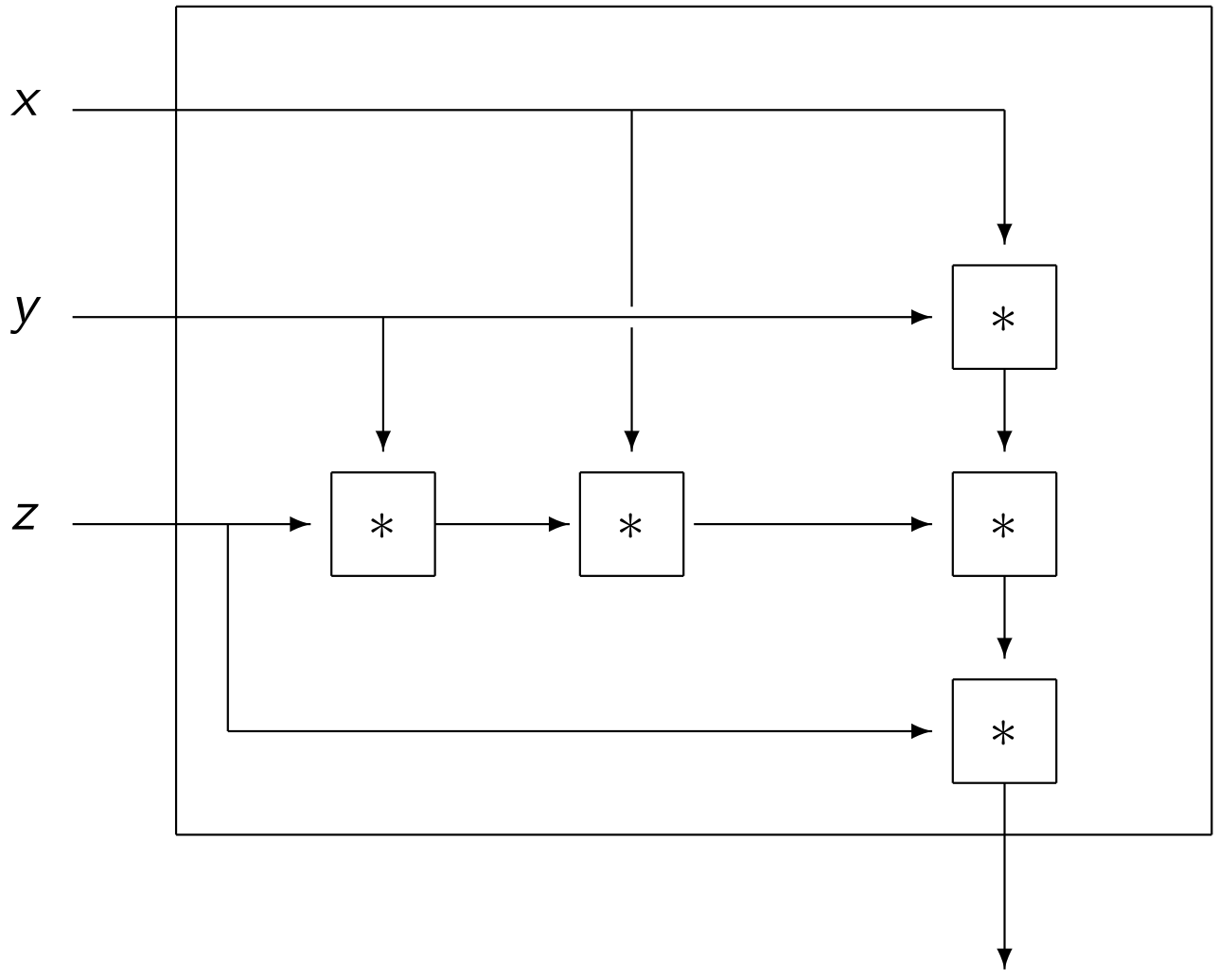


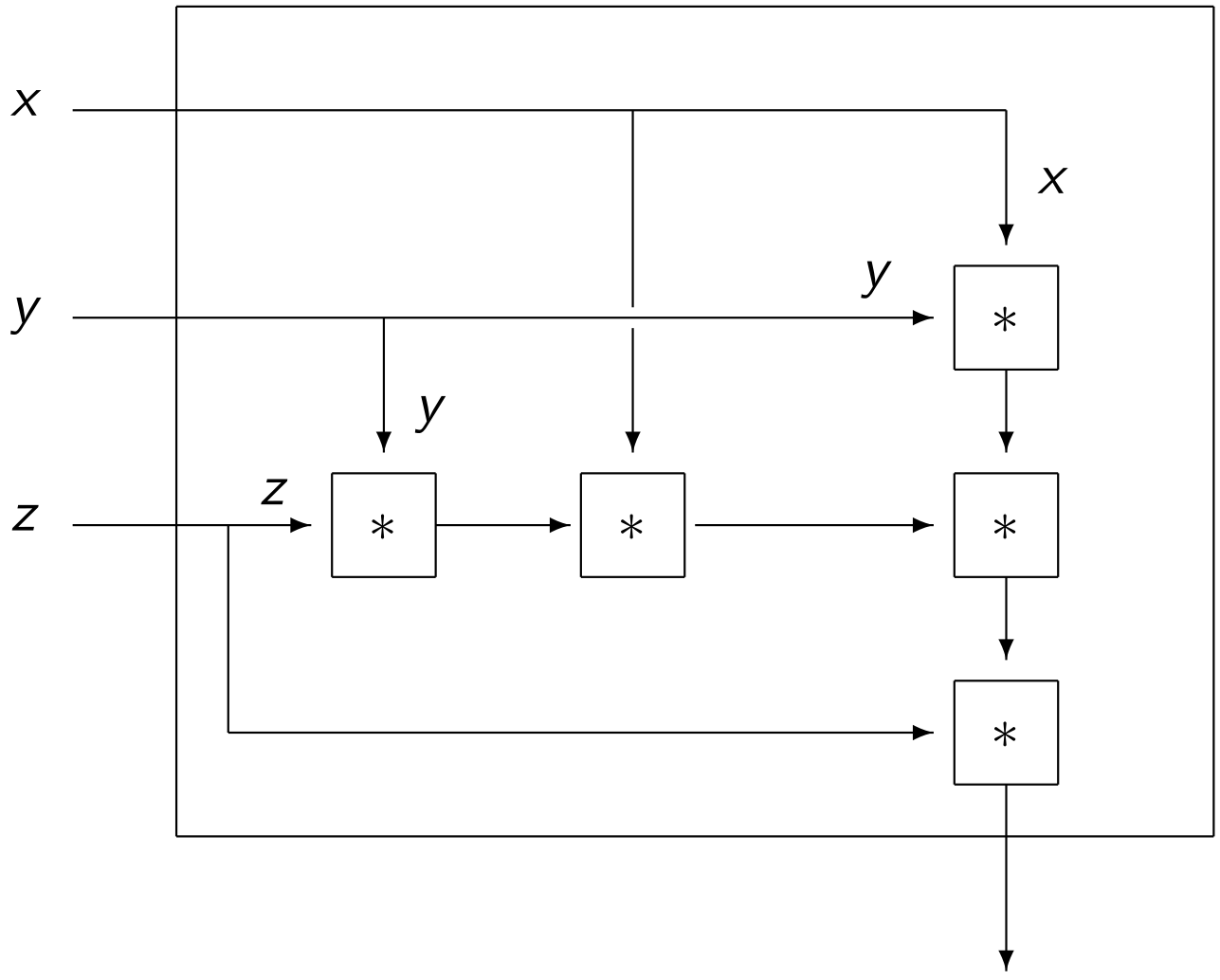
x	y	z	Output
0	0	0	3
0	0	1	0
0	0	2	2
0	0	3	1
0	1	0	2
0	1	1	0
0	1	2	3
0	1	3	3
0	2	0	1
0	2	1	2
0	2	2	0
:	:	:	:
3	3	0	0
3	3	1	3
3	3	2	1
3	3	3	2

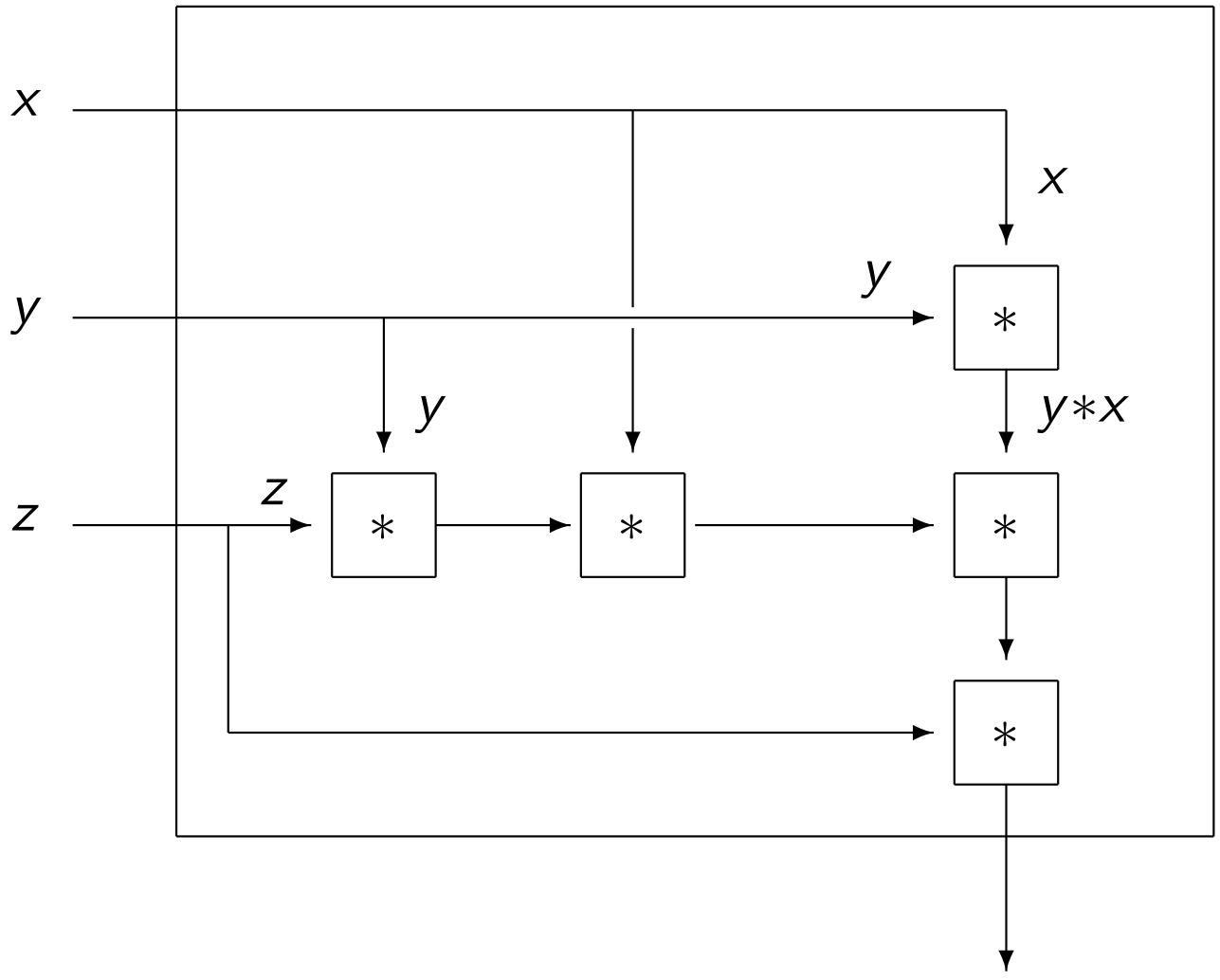
$*$	0	1	2	3
0	3	0	0	3
1	2	2	1	0
2	3	0	2	0
3	2	0	1	3

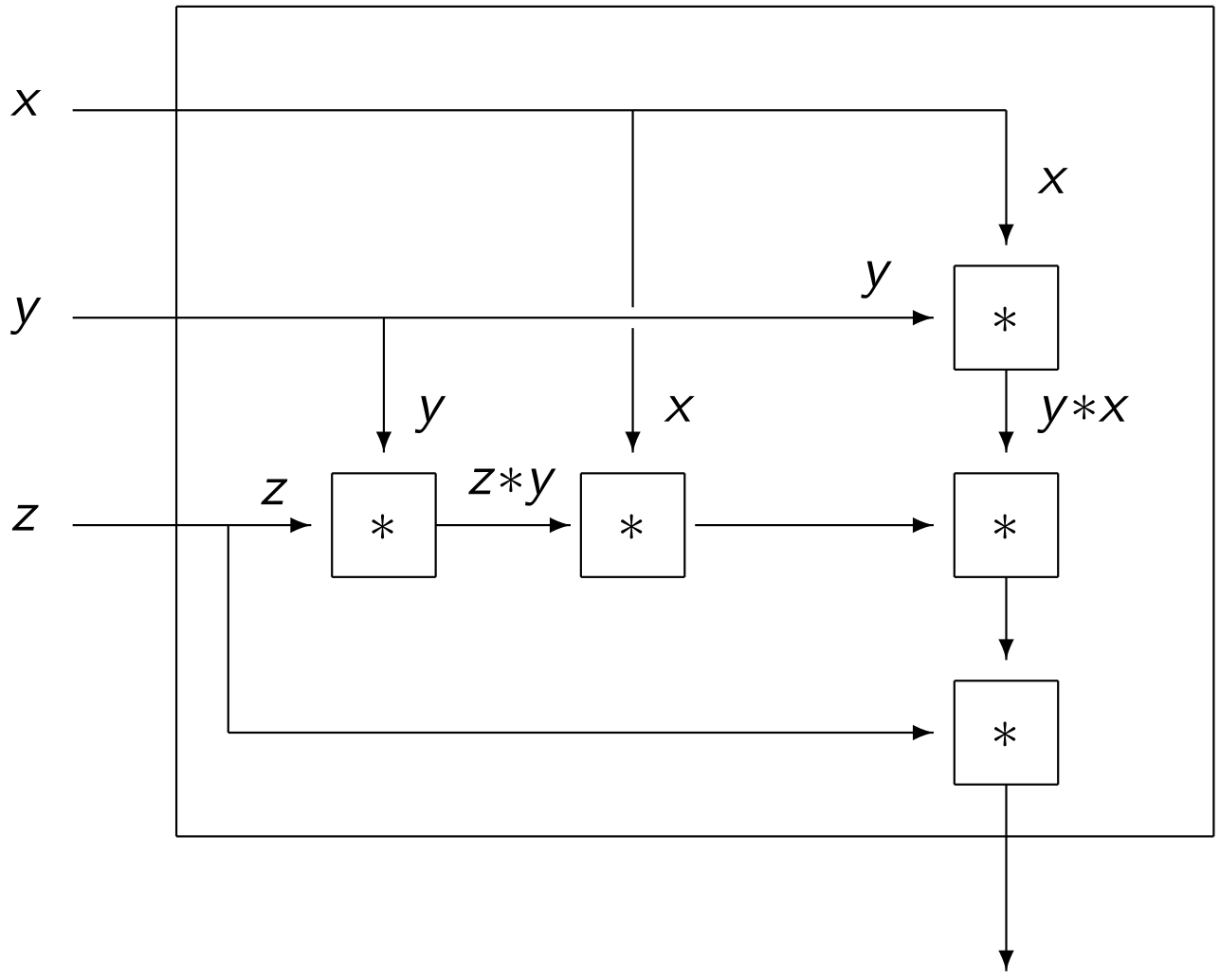
Switching Circuit

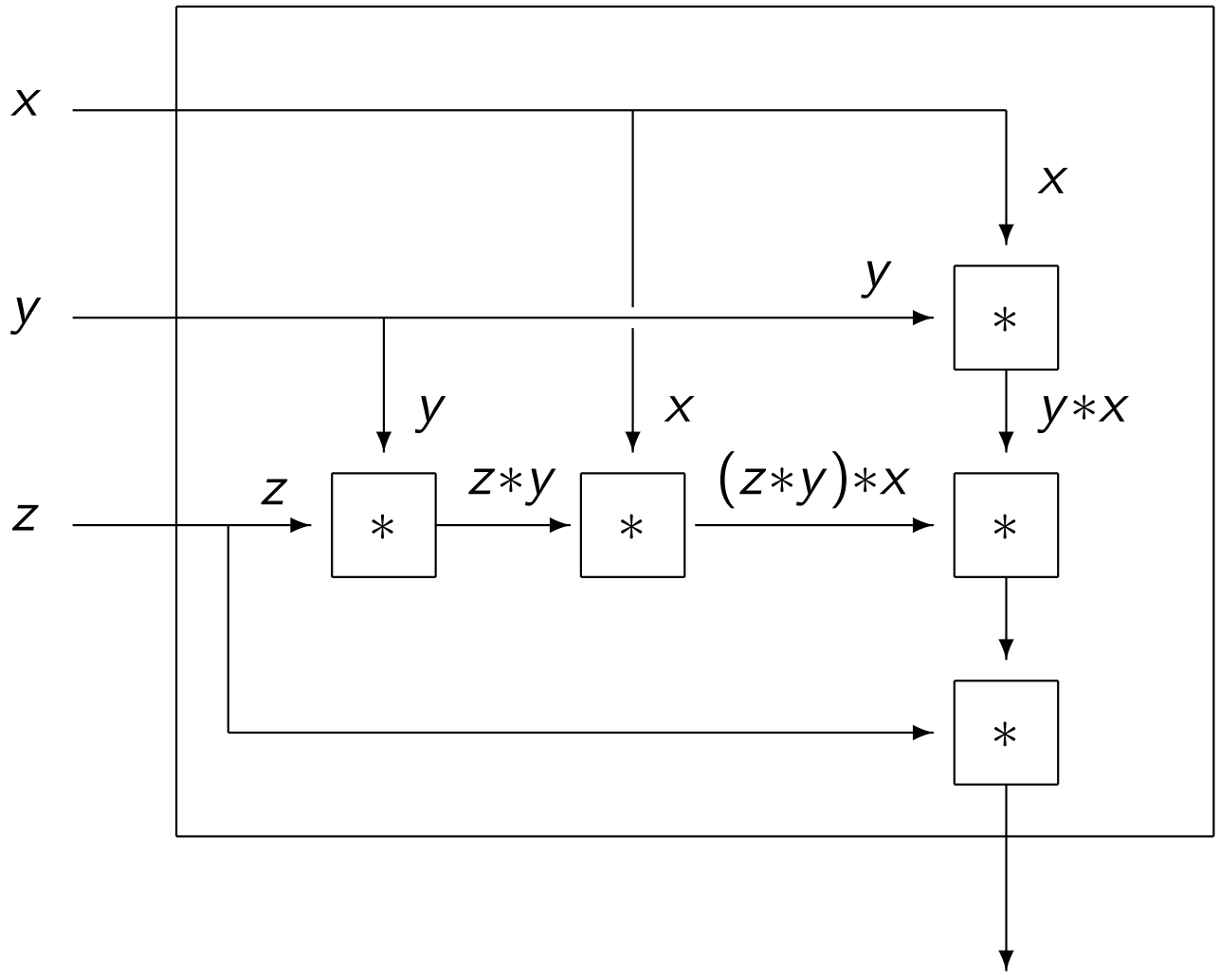


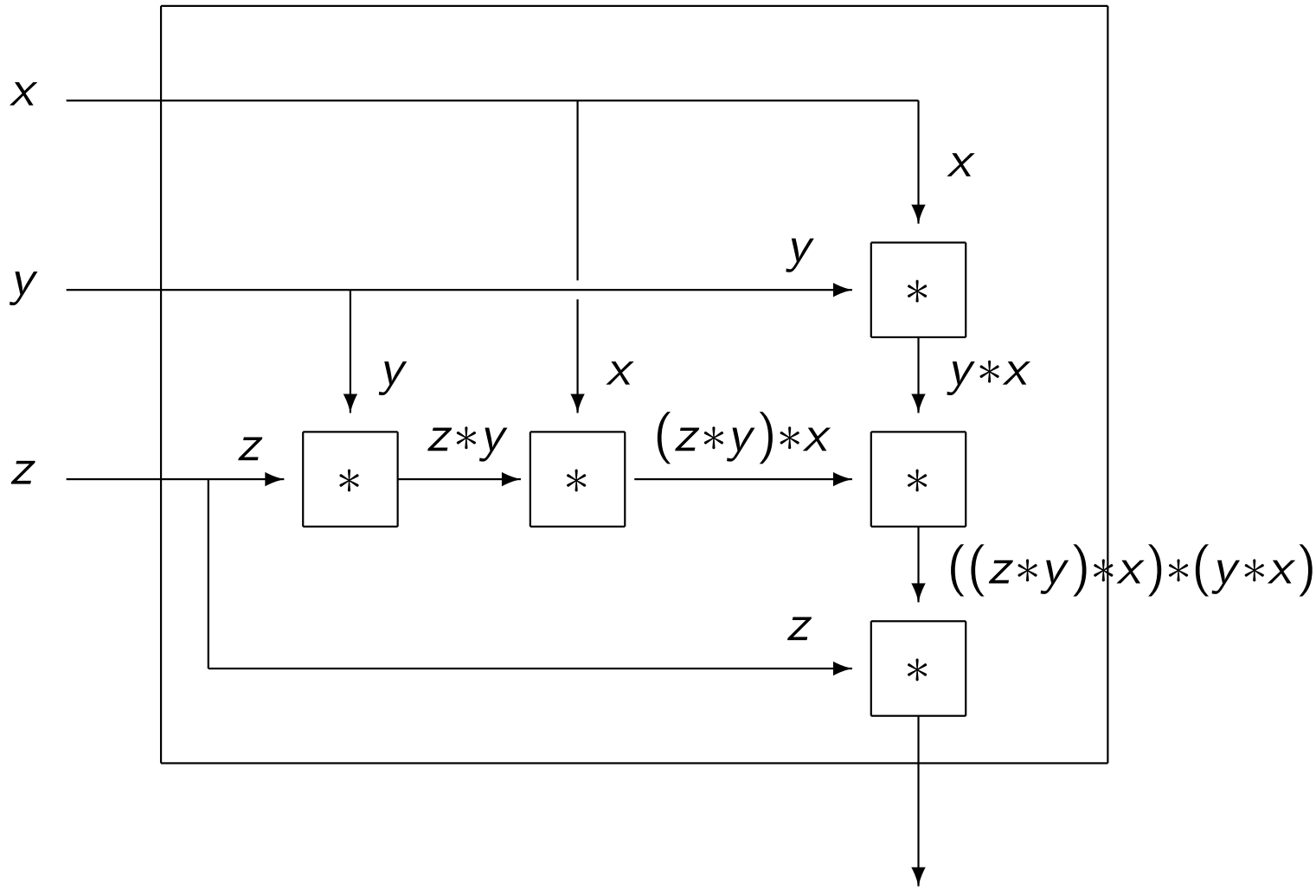

x
y
z
*
*
*
*
*

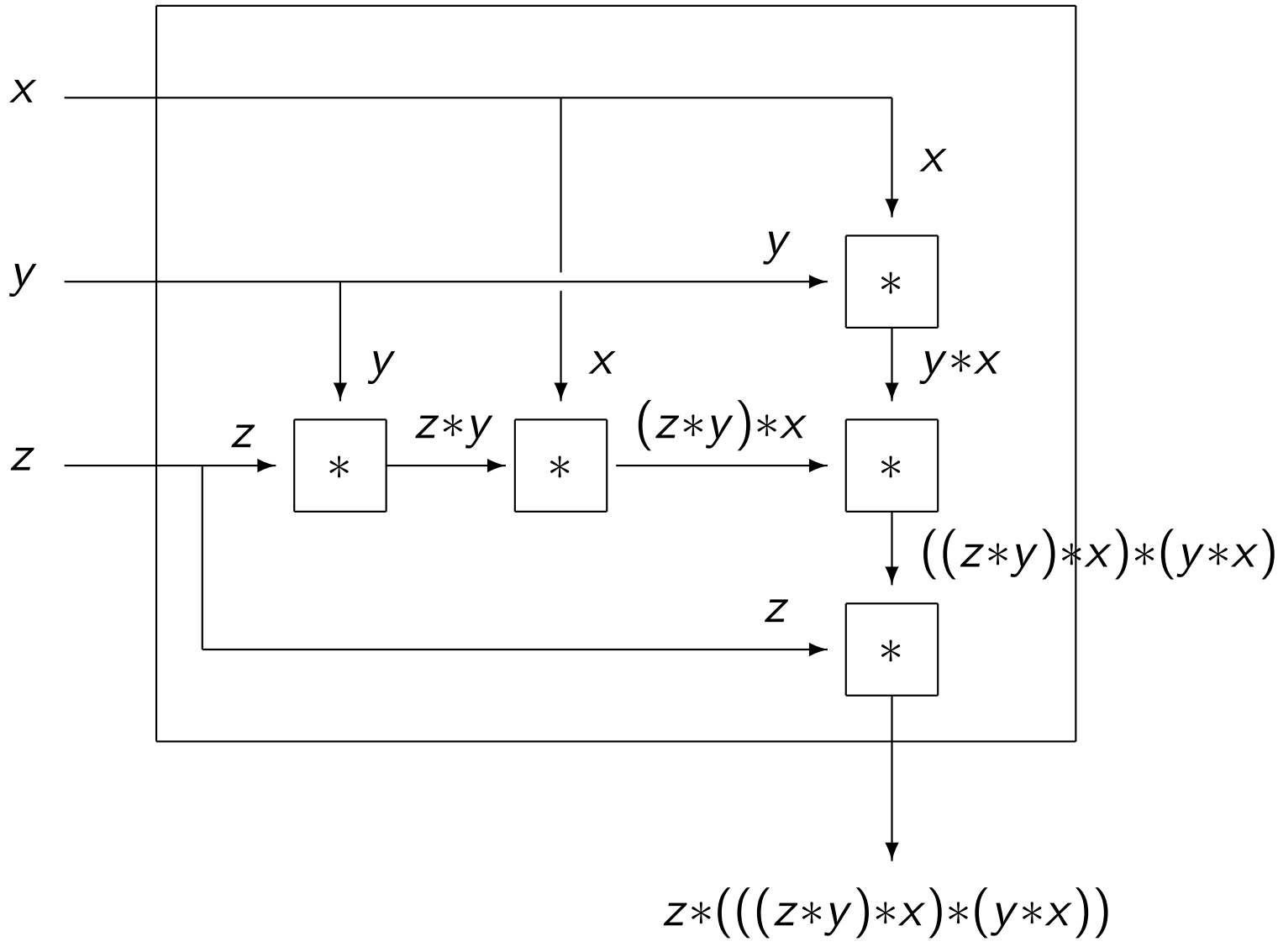

x
y
z
x
y
y
z
*
*
*
*
*


x
y
z
x
y
y
z
*
*
*
*
*
y∗x


x
y
z
x
y
*
y
x
y∗x
z
*
z∗y
*
*
*

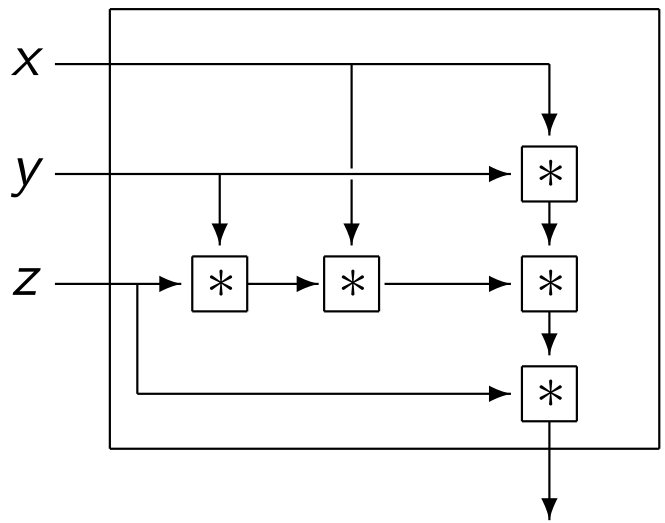

x
y
z
x
y
y
x
y∗x
z
z∗y
(z∗y)∗x
∗
∗
∗
∗
∗


x
y
z
x
y
y
x
z
z∗y
(z∗y)∗x
y∗x
((z∗y)∗x)∗(y∗x)
z
∗
∗
∗
∗
∗


x
y
z
x
y
y
x
z
z∗y
(z∗y)∗x
y∗x
((z∗y)∗x)∗(y∗x)
z
z∗(((z∗y)∗x)∗(y∗x))
∗
∗
∗
∗

The expression $z*(((z*y)*x)*(y*x))$ is both

- the circuit output if its inputs are x, y and z and the gate operation is $*$ and
- a complete design or blueprint for the circuit itself.



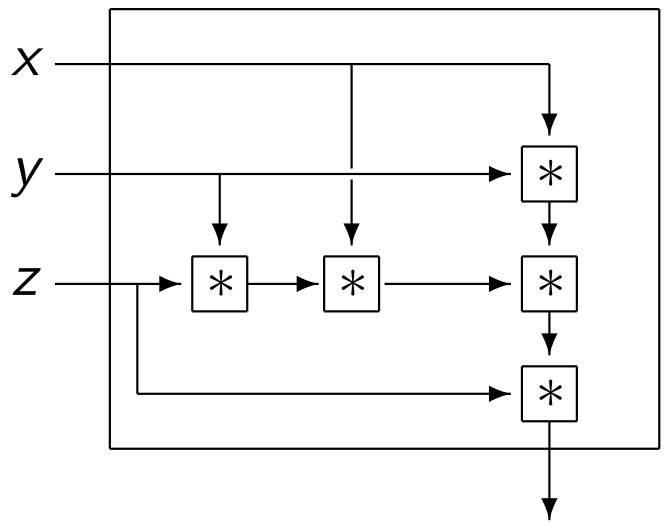
The expression $z*(((z*y)*x)*(y*x))$ is both

- the circuit output if its inputs are x, y and z and the gate operation is $*$ and
- a complete design or blueprint for the circuit itself.

We will call it the

DNA

for the circuit.

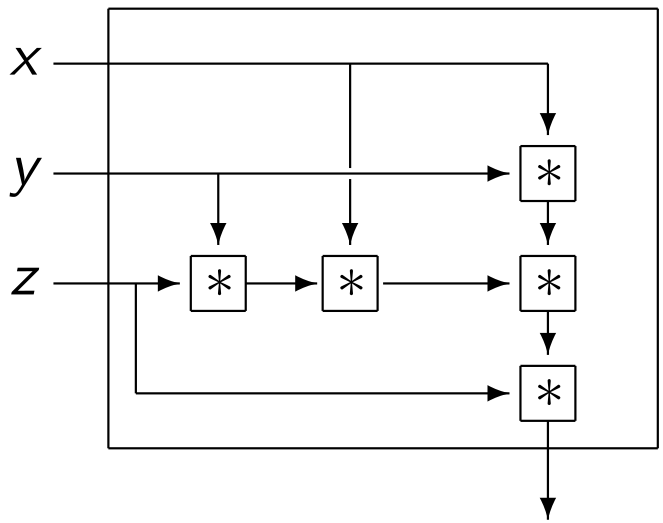


3. Design Problem

x	y	z	Output
0	0	0	
0	0	1	
0	0	2	
0	1	0	
0	1	1	
0	1	2	
0	2	0	
0	2	1	???
0	2	2	
1	0	0	
1	0	1	
:	:	:	:
2	1	2	
2	2	0	
2	2	1	
2	2	2	

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

$\Longleftarrow$

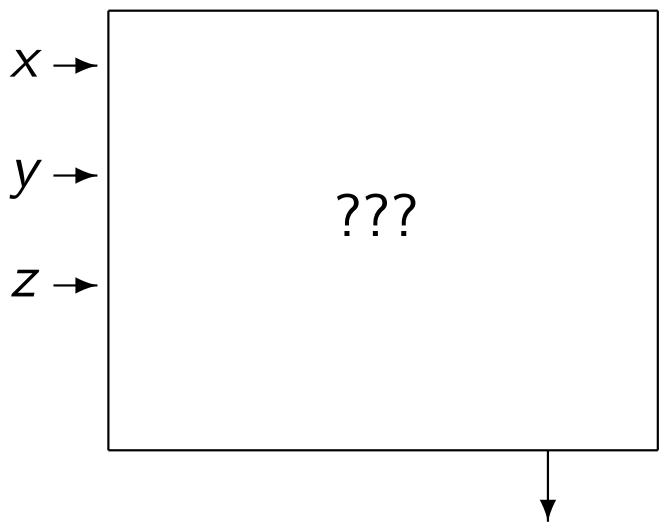


x	y	z	Target
0	0	0	2
0	0	1	1
0	0	2	0
0	1	0	2
0	1	1	1
0	1	2	0
0	2	0	2
0	2	1	1
0	2	2	0
1	0	0	1
1	0	1	2
:	:	:	:
2	1	2	0
2	2	0	2
2	2	1	1
2	2	2	0

$\Longrightarrow$

Design Problem

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

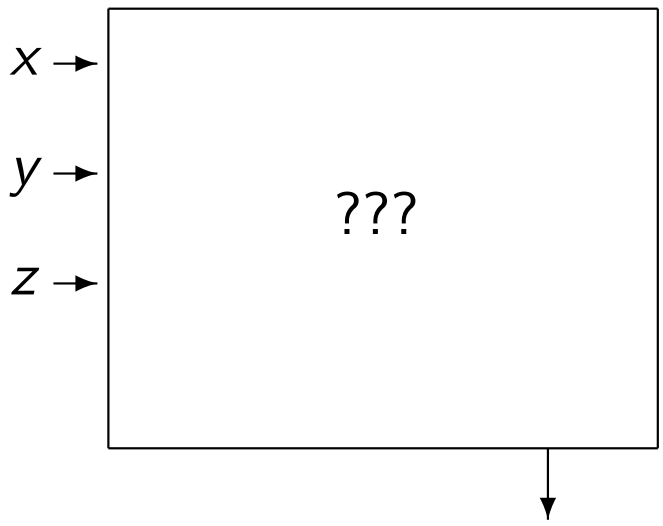


x	y	z	Target
0	0	0	2
0	0	1	1
0	0	2	0
0	1	0	2
0	1	1	1
0	1	2	0
0	2	0	2
0	2	1	1
0	2	2	0
1	0	0	1
1	0	1	2
:	:	:	:
2	1	2	0
2	2	0	2
2	2	1	1
2	2	2	0

Design Problem

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

$\Longrightarrow$



- The number of different possible target sequences is $3^{27} \approx 10^{13}$.

x	y	z	Target
0	0	0	0
0	0	1	0
0	0	2	2
0	1	0	0
0	1	1	1
0	1	2	2
0	2	0	1
0	2	1	0
0	2	2	1
1	0	0	2
1	0	1	0
:	:	:	:
2	1	2	0
2	2	0	1
2	2	1	1
2	2	2	0

Design Problem

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

- A fast computer can solve the design problem by a **random search**, generating and testing about a million circuits per second until a solution is found.

x	y	z	Target
0	0	0	0
0	0	1	0
0	0	2	2
0	1	0	0
0	1	1	1
0	1	2	2
0	2	0	1
0	2	1	0
0	2	2	1
1	0	0	2
1	0	1	0
:	:	:	:
2	1	2	0
2	2	0	1
2	2	1	1
2	2	2	0

Design Problem

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

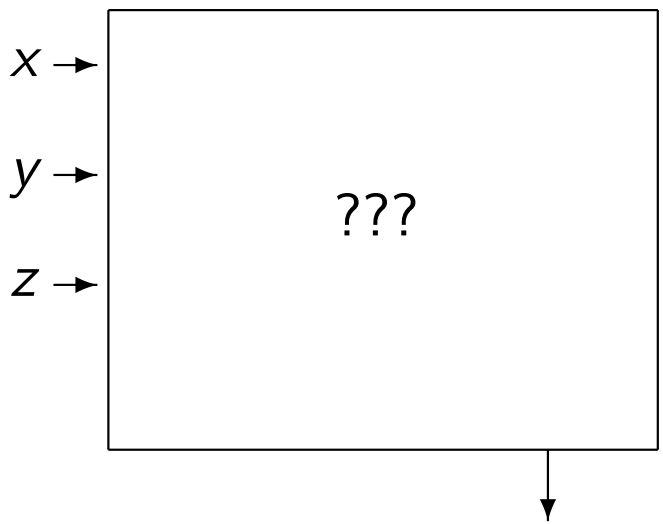
- A fast computer can solve the design problem by a **random search**, generating and testing about a million circuits per second until a solution is found.
- The expected time to find a solution would be $10^{13}/10^{6} \approx 10^{7}$ seconds, or about **4 months**.

4. One Parent Evolution

— Lee Spector

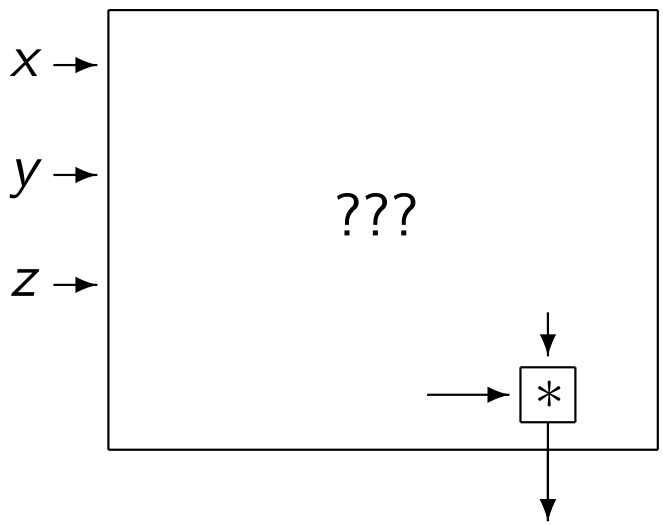
x	y	z	Target
0	0	0	2
0	0	1	1
0	0	2	2
0	1	0	1
0	1	1	0
0	1	2	2
0	2	0	0
0	2	1	2
0	2	2	0
1	0	0	2
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	2
2	2	1	0
2	2	2	1

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1



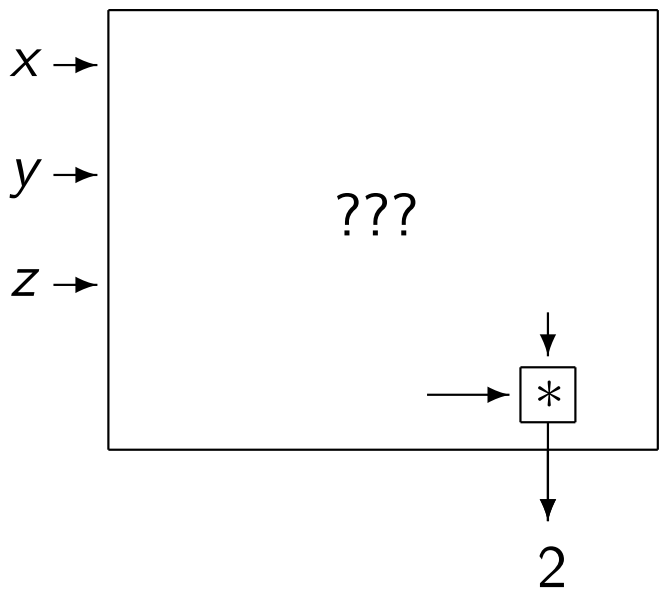
x	y	z	Target
0	0	0	2
0	0	1	1
0	0	2	2
0	1	0	1
0	1	1	0
0	1	2	2
0	2	0	0
0	2	1	2
0	2	2	0
1	0	0	2
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	2
2	2	1	0
2	2	2	1

$*$	0	1	2
0	2	2	2
1	2	2	2
2	2	2	2



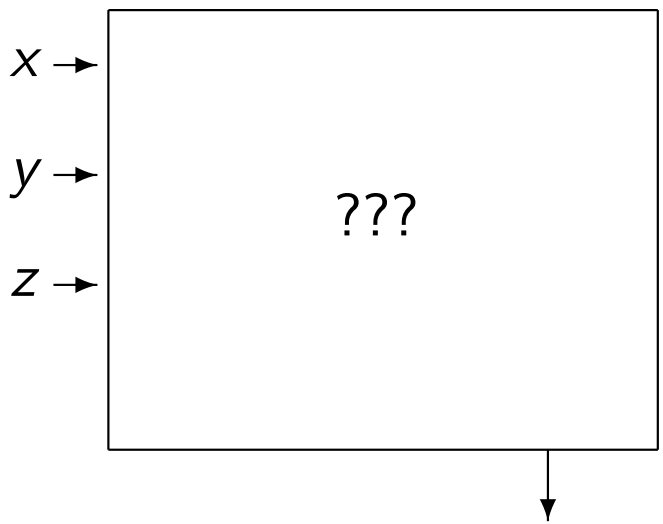
x	y	z	Target
0	0	0	2
0	0	1	1
0	0	2	2
0	1	0	1
0	1	1	0
0	1	2	2
0	2	0	0
0	2	1	2
0	2	2	0
1	0	0	2
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	2
2	2	1	0
2	2	2	1

$*$	0	1	2
0	2	2	2
1	2	2	2
2	2	2	2



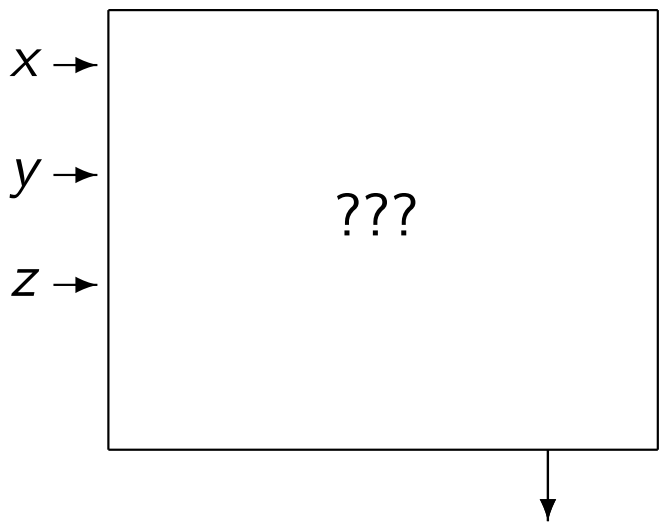
x	y	z	Target
0	0	0	2
0	0	1	1
0	0	2	2
0	1	0	1
0	1	1	0
0	1	2	2
0	2	0	0
0	2	1	2
0	2	2	0
1	0	0	2
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	2
2	2	1	0
2	2	2	1

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1



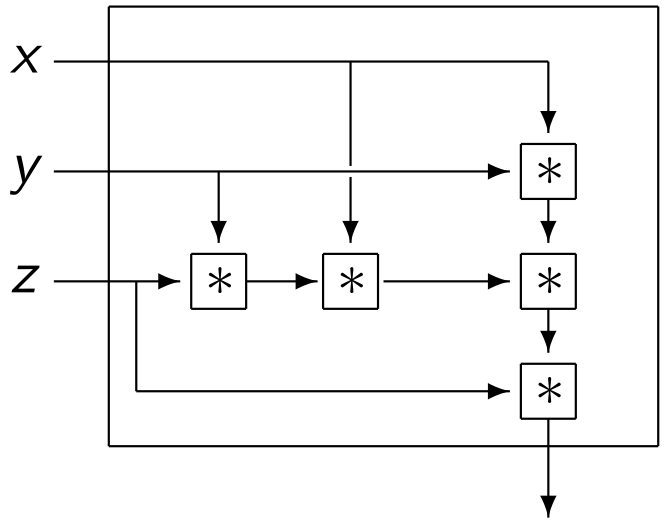
x	y	z	Target
0	0	0	2
0	0	1	1
0	0	2	2
0	1	0	1
0	1	1	0
0	1	2	2
0	2	0	0
0	2	1	2
0	2	2	0
1	0	0	2
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	2
2	2	1	0
2	2	2	1

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

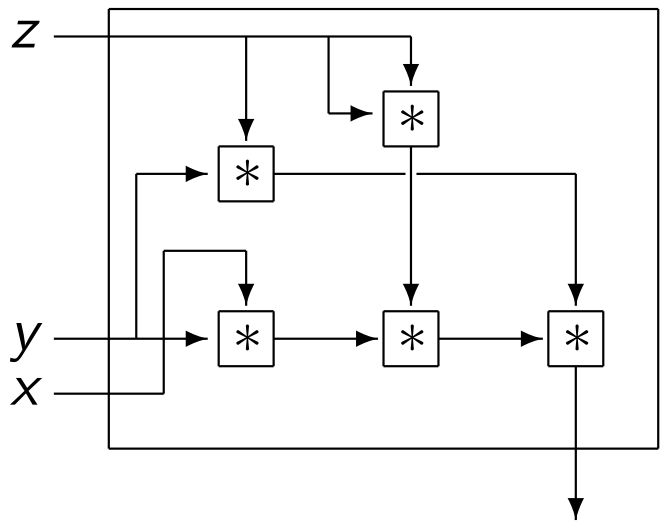


x	y	z	Target	SC1	SC2
0	0	0	2	2	0
0	0	1	1	1	1
0	0	2	2	0	2
0	1	0	1	2	1
0	1	1	0	1	1
0	1	2	2	0	2
0	2	0	0	2	0
0	2	1	2	1	0
0	2	2	0	0	0
1	0	0	2	1	2
1	0	1	0	0	1
:	:	:	:	:	:
2	1	2	2	0	2
2	2	0	2	2	2
2	2	1	0	1	1
2	2	2	1	0	1

Switching Circuit 1



Switching Circuit 2



x	y	z	Target	SC1	SC2
0	0	0	2	2	0
0	0	1	1	1	1
0	0	2	2	0	2
0	1	0	1	2	1
0	1	1	0	1	1
0	1	2	2	0	2
0	2	0	0	2	0
0	2	1	2	1	0
0	2	2	0	0	0
1	0	0	2	1	2
1	0	1	0	0	1
:	:	:	:	:	:
2	1	2	2	0	2
2	2	0	2	2	2
2	2	1	0	1	1
2	2	2	1	0	1

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

x	y	z	Target	SC1	SC2
0	0	0	2	2	0
0	0	1	1	1	1
0	0	2	2	0	2
0	1	0	1	2	1
0	1	1	0	1	1
0	1	2	2	0	2
0	2	0	0	2	0
0	2	1	2	1	0
0	2	2	0	0	0
1	0	0	2	1	2
1	0	1	0	0	1
:	:	:	:	:	:
2	1	2	2	0	2
2	2	0	2	2	2
2	2	1	0	1	1
2	2	2	1	0	1

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Given any target and gate, we define the **fitness** of a circuit to be the number of its outputs that match the target.

x	y	z	Target	SC1	SC2
0	0	0	2	2	0
0	0	1	1	1	1
0	0	2	2	0	2
0	1	0	1	2	1
0	1	1	0	1	1
0	1	2	2	0	2
0	2	0	0	2	0
0	2	1	2	1	0
0	2	2	0	0	0
1	0	0	2	1	2
1	0	1	0	0	1
:	:	:	:	:	:
2	1	2	2	0	2
2	2	0	2	2	2
2	2	1	0	1	1
2	2	2	1	0	1

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Given any target and gate, we define the **fitness** of a circuit to be the number of its outputs that match the target.

fitness of SC1 ≈ 9
fitness of SC2 ≈ 18

x	y	z	Target	SC1	SC2
0	0	0	2	2	0
0	0	1	1	1	1
0	0	2	2	0	2
0	1	0	1	2	1
0	1	1	0	1	1
0	1	2	2	0	2
0	2	0	0	2	0
0	2	1	2	1	0
0	2	2	0	0	0
1	0	0	2	1	2
1	0	1	0	0	1
:	:	:	:	:	:
2	1	2	2	0	2
2	2	0	2	2	2
2	2	1	0	1	1
2	2	2	1	0	1

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Given any target and gate, we define the **fitness** of a circuit to be the number of its outputs that match the target.

fitness of SC1 ≈ 9
fitness of SC2 ≈ 18

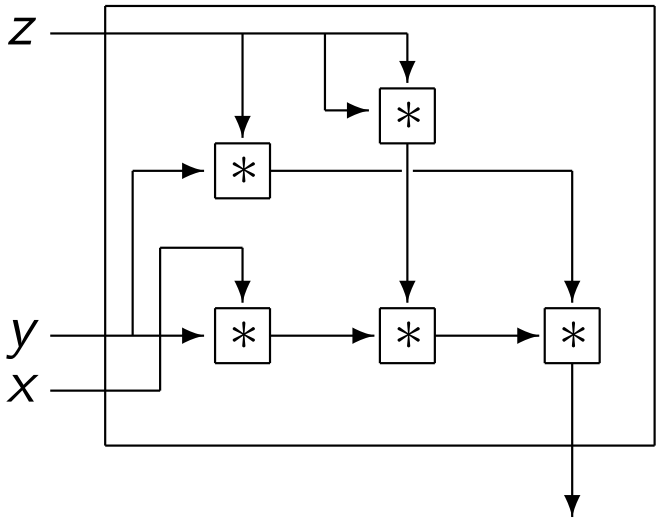
A solution to the Design Problem will be any switching circuit with a fitness of 27.

$$((y * x) * (z * z)) * (y * z)$$

DNA Sequences

$$((y * x) * (z * z)) * (y * z)$$

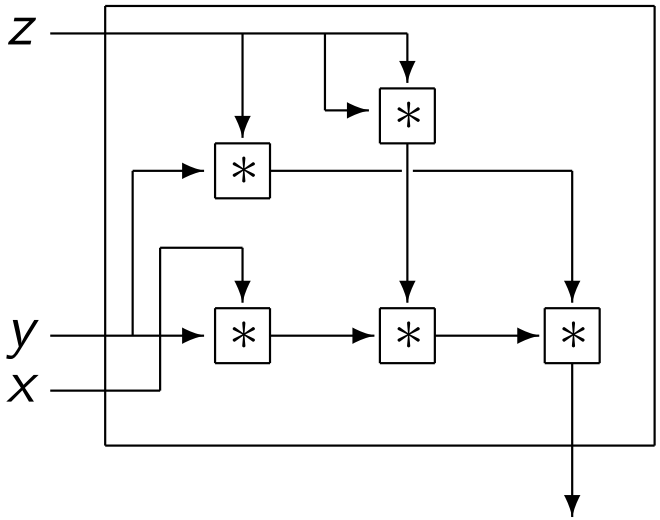
DNA Sequences
(Genotypes)



Physical Switching Circuits
(Phenotypes)

$\longleftarrow ((y * x) * (z * z)) * (y * z)$

DNA Sequences
(Genotypes)



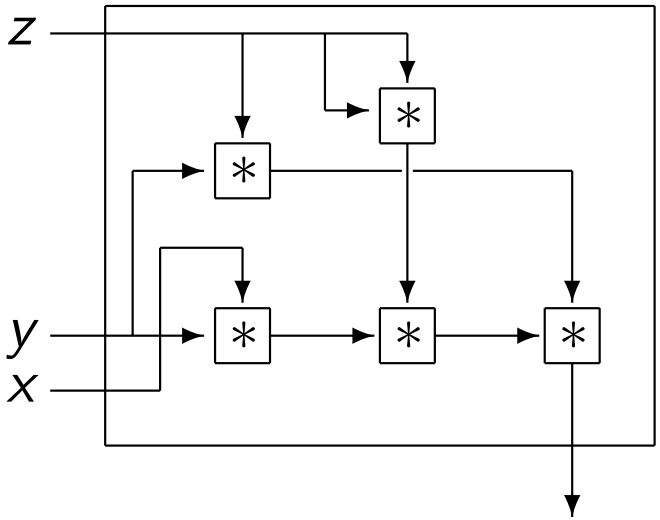
Physical Switching Circuits
(Phenotypes)

$\leftarrow ((y * x) * (z * z)) * (y * z)$

DNA Sequences
(Genotypes)

$\downarrow$

Calculate Fitnesses



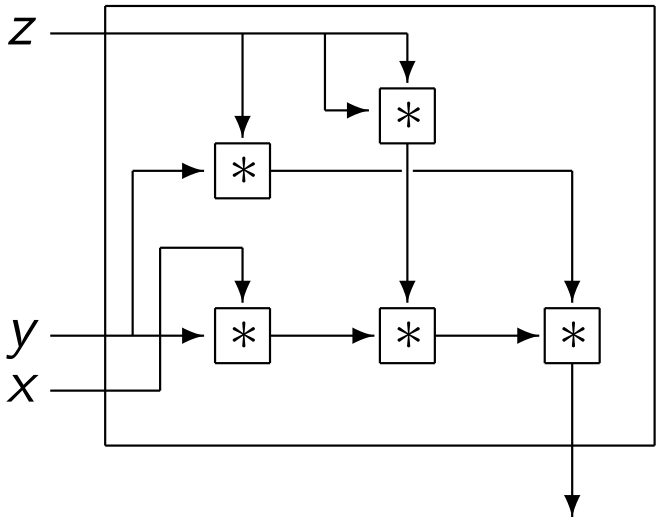
Physical Switching Circuits
(Phenotypes)

$\longleftarrow$ $((y * x) * (z * z)) * (y * z)$

DNA Sequences
(Genotypes)

$\downarrow$

Calculate Fitnesses $\longrightarrow$ Generate DNA for the Fittest Circuits



Physical Switching Circuits
(Phenotypes)

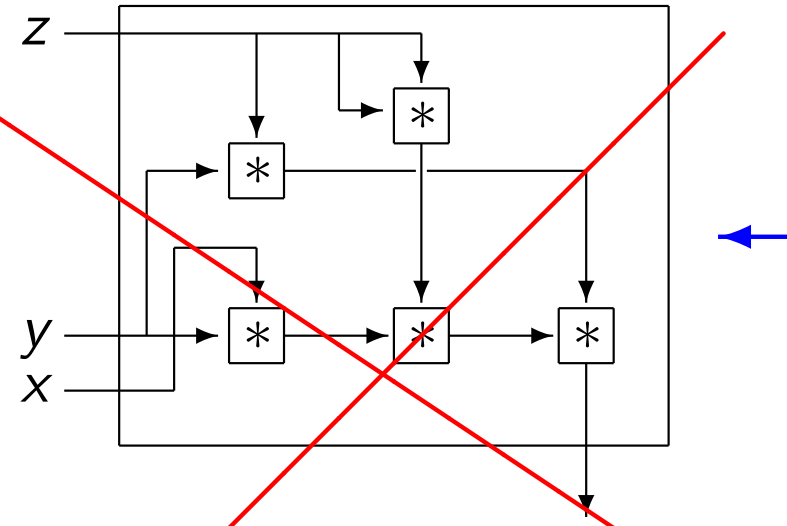
$((y * x) * (z * z)) * (y * z)$

DNA Sequences
(Genotypes)

Add
Variations

Calculate Fitnesses

Generate DNA for the
Fittest Circuits



Physical Switching Circuits
(Phenotypes)

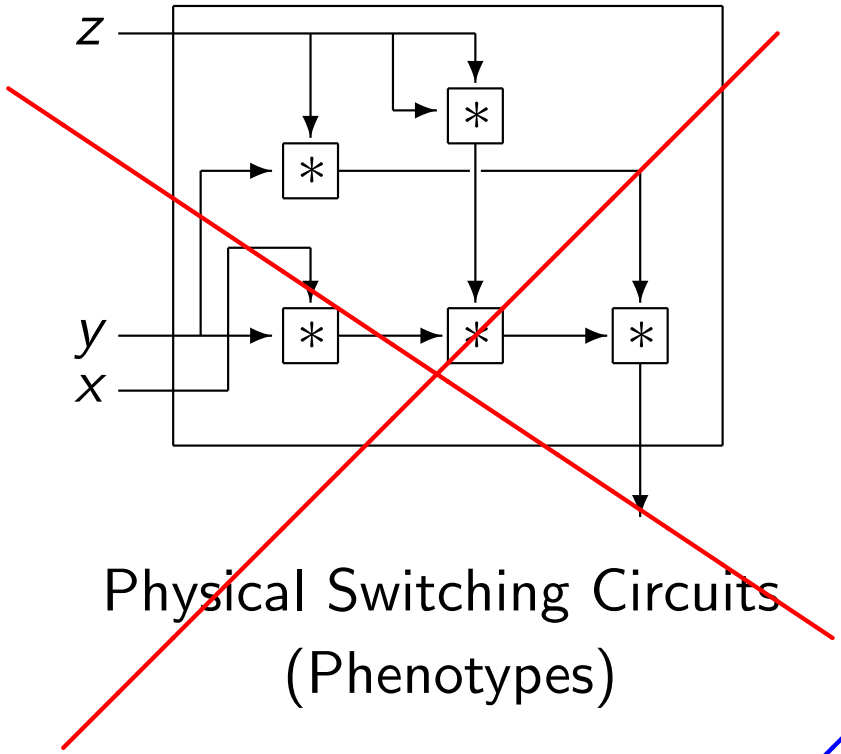
$((y * x) * (z * z)) * (y * z)$

DNA Sequences
(Genotypes)

Add
Variations

Calculate Fitnesses

Generate DNA for the
Fittest Circuits



Physical Switching Circuits
(Phenotypes)

$((y * x) * (z * z)) * (y * z)$

DNA Sequences
(Genotypes)

Add
Variations

Calculate Fitnesses → Generate DNA for the Fittest Circuits

x	y	z	Target
0	0	0	0
0	0	1	1
0	0	2	2
0	1	0	0
0	1	1	0
0	1	2	0
0	2	0	0
0	2	1	0
0	2	2	0
1	0	0	1
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	0
2	2	1	1
2	2	2	2

Possible target sequences:
$3^{27} \approx 10^{13}$

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

x	y	z	Target
0	0	0	0
0	0	1	1
0	0	2	2
0	1	0	0
0	1	1	0
0	1	2	0
0	2	0	0
0	2	1	0
0	2	2	0
1	0	0	1
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	0
2	2	1	1
2	2	2	2

Possible target sequences:
$3^{27} \approx 10^{13}$

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Typical solution to design problem, generated by **random search:**

x	y	z	Target
0	0	0	0
0	0	1	1
0	0	2	2
0	1	0	0
0	1	1	0
0	1	2	0
0	2	0	0
0	2	1	0
0	2	2	0
1	0	0	1
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	0
2	2	1	1
2	2	2	2

Possible target sequences:
$3^{27} \approx 10^{13}$

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Typical solution to design problem, generated by **random search: 4 months**

x	y	z	Target
0	0	0	0
0	0	1	1
0	0	2	2
0	1	0	0
0	1	1	0
0	1	2	0
0	2	0	0
0	2	1	0
0	2	2	0
1	0	0	1
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	0
2	2	1	1
2	2	2	2

Possible target sequences:
$3^{27} \approx 10^{13}$

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Typical solution to design problem, generated

by **random search: 4 months**

by **evolutionary computation:**

x	y	z	Target
0	0	0	0
0	0	1	1
0	0	2	2
0	1	0	0
0	1	1	0
0	1	2	0
0	2	0	0
0	2	1	0
0	2	2	0
1	0	0	1
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	0
2	2	1	1
2	2	2	2

Possible target sequences:
$3^{27} \approx 10^{13}$

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Typical solution to design problem, generated

by **random search: 4 months**

by **evolutionary computation: 5 minutes**

$$((((((((x*(y*x))*x)*z)*(z*x))*((x*(z*(x*(z*y))))*z))*z)*z)*(z*((((x*(((z*z)*x)*(z*x)))*x)*y)*(((y*(z*(z*y)))*(((y*y)*x)*z))*(x*(((z*z)*x)*(z*(x*(z*y))))))))$$

x	y	z	Target
0	0	0	0
0	0	1	1
0	0	2	2
0	1	0	0
0	1	1	0
0	1	2	0
0	2	0	0
0	2	1	0
0	2	2	0
1	0	0	1
1	0	1	0
:	:	:	:
2	1	2	2
2	2	0	0
2	2	1	1
2	2	2	2

Possible target sequences:
$3^{27} \approx 10^{13}$

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Typical solution to design problem, generated

by **random search: 4 months**

by **evolutionary computation: 5 minutes**

$$((((((((x*(y*x))*x)*z)*(z*x))*((x*(z*(x*(z*y))))*z))*z)*z)*(z*((((x*(((z*z)*x)*(z*x)))*x)*y)*(((y*(z*(z*y)))*(((y*y)*x)*z))*(x*(((z*z)*x)*(z*(x*(z*y))))))))$$

(Won the $5000 gold metal award in the 2008 International HUMIE Competition sponsored by the Association for Computing Machinery.)

5. Two Parent Evolution

— Lee Spector

— Nick Falco

A gate is **term continuous** if small changes in a circuit built with that gate result in small changes it its output sequence.

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1. D. Clark, Evolution of algebraic terms 1: term to term operation continuity, *International Journal of Algebra and Computation*, Vol. 23, No. 5 (2013) 1175-1205.

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Theorem. *A gate is term continuous if and only if it is AC and has NSR.*

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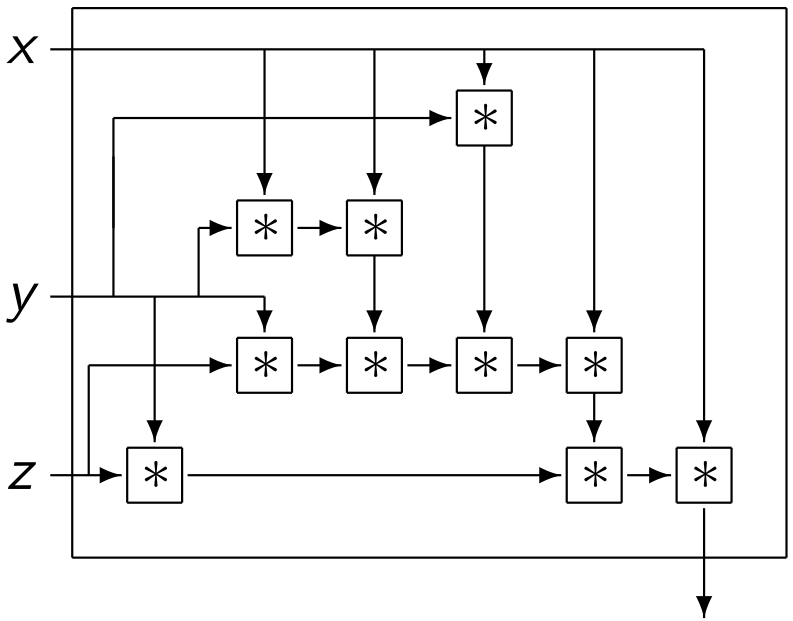
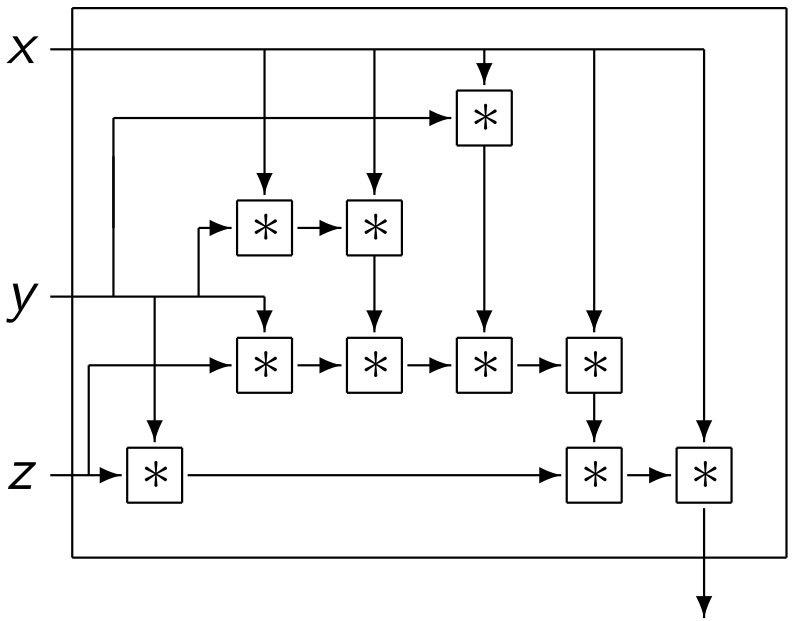
1. D. Clark, Evolution of algebraic terms 1: term to term operation continuity, *International Journal of Algebra and Computation*, Vol. 23, No. 5 (2013) 1175-1205.

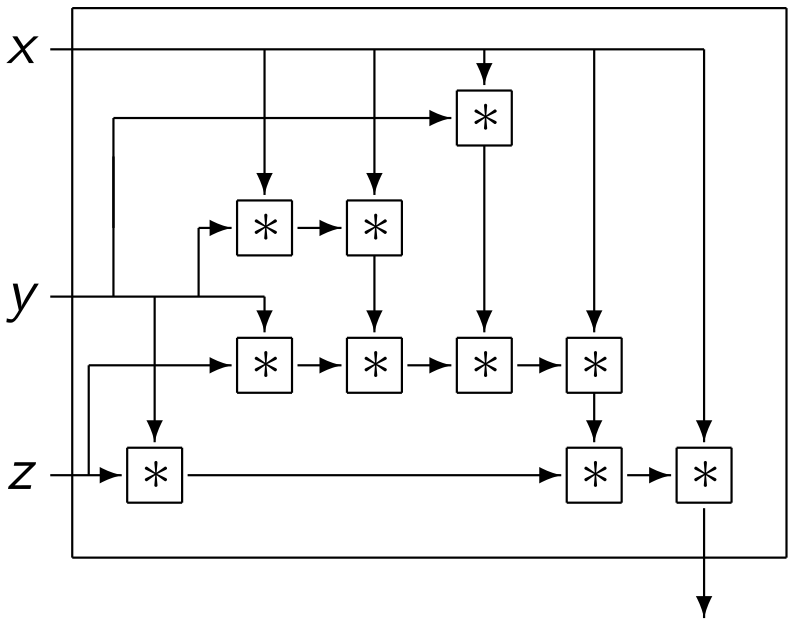
Theorem. *A gate is term continuous if and only if it is AC and has NSR.*



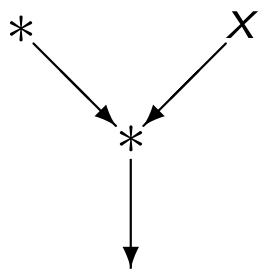
$$\Big[(z*y)*\Big(\Big(\big[(z*y)*((y*x)*x)\big]*(y*x)\Big)*x\Big)\Big]*x$$

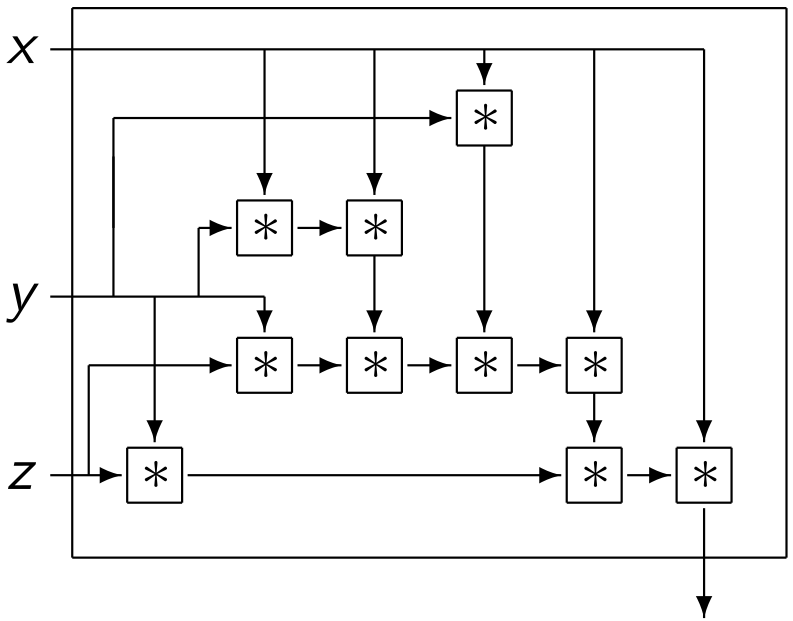


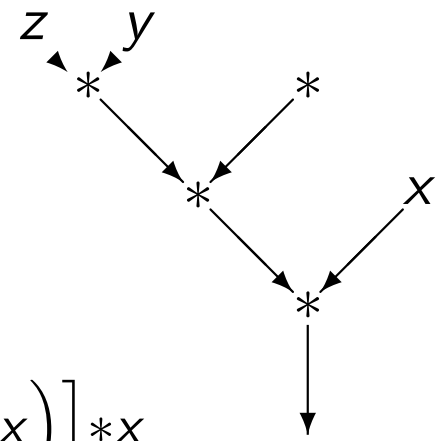
$$\left[(z*y)*\left(\left(\left[(z*y)*((y*x)*x)\right]*(y*x)\right)*x\right)\right]*x$$



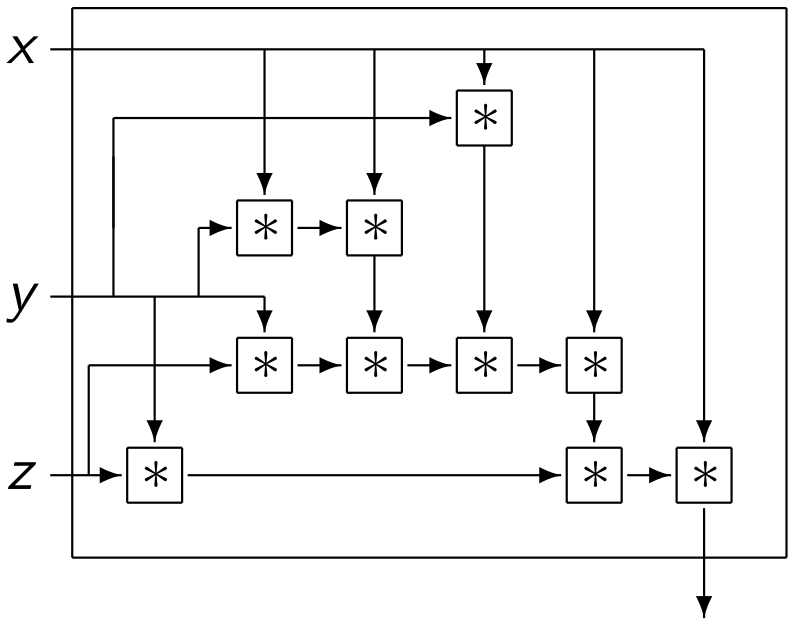
$$\left[(z*y)*\left(\left(\left[(z*y)*((y*x)*x)\right]*(y*x)\right)*x\right)\right]*x$$

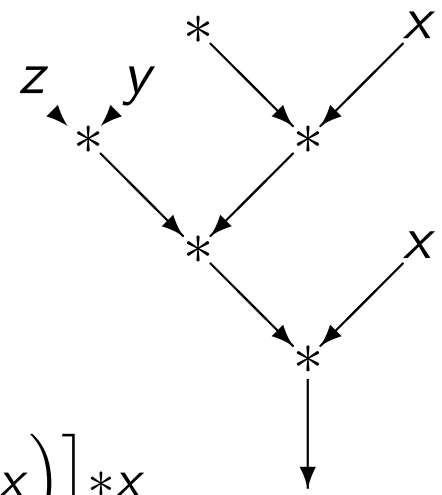




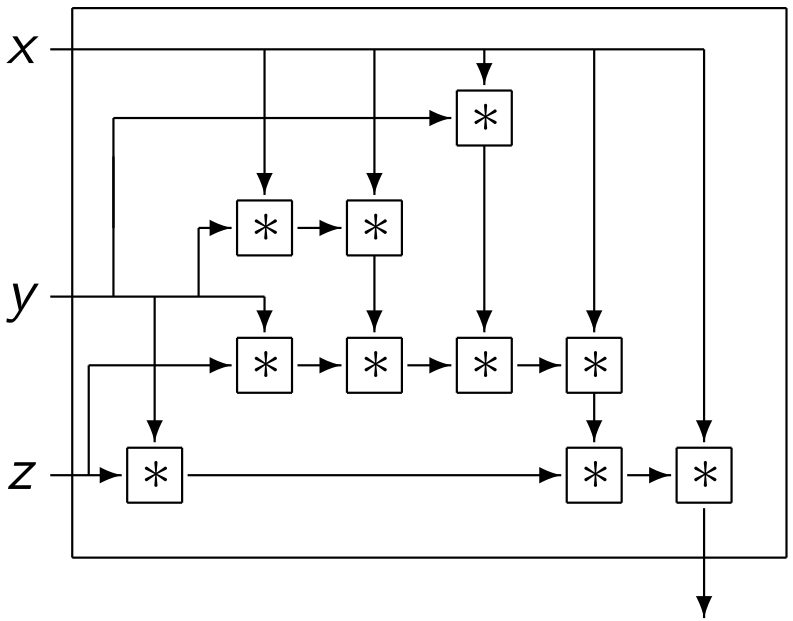


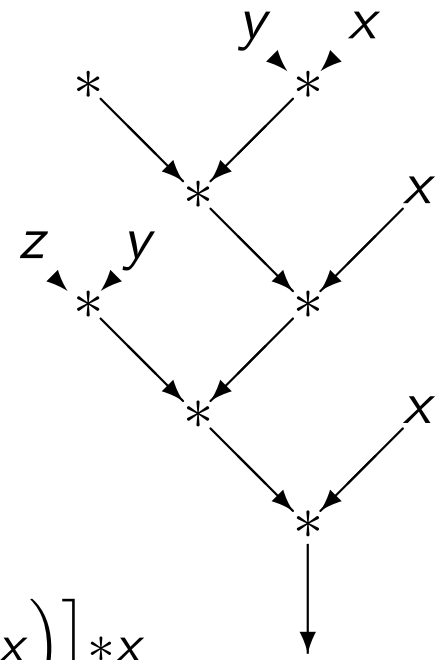
$$\Big[(z*y)*\Big(\Big(\big[(z*y)*((y*x)*x)\big]*(y*x)\Big)*x\Big)\Big]*x$$



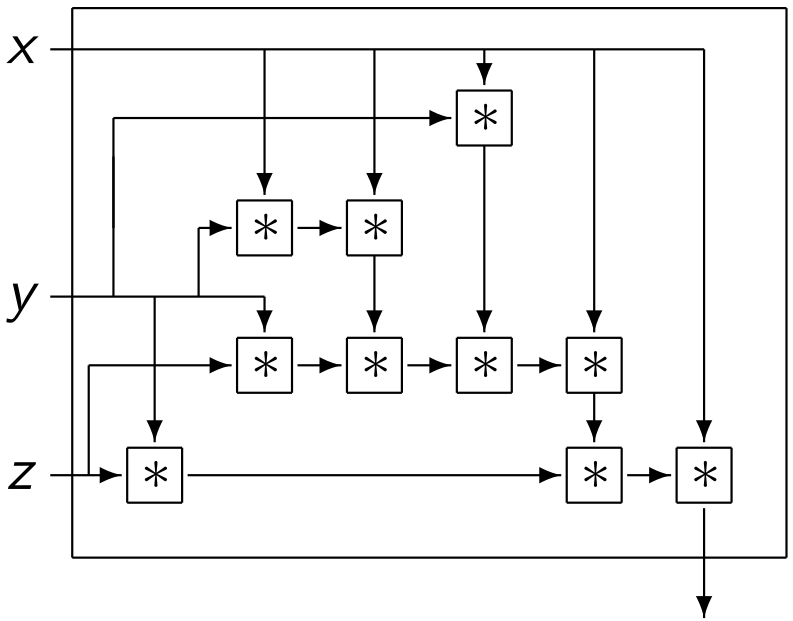


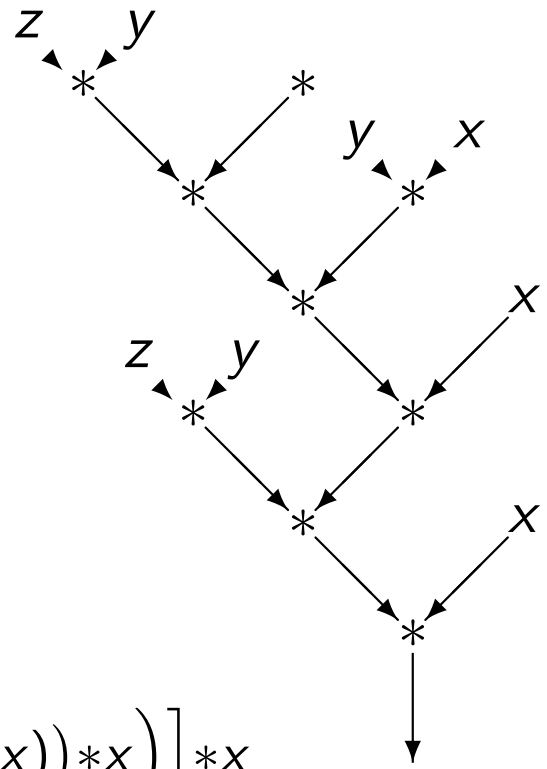
$$\Big[(z*y)*\Big(\Big(\big[(z*y)*((y*x)*x)\big]*(y*x)\Big)*x\Big)\Big]*x$$



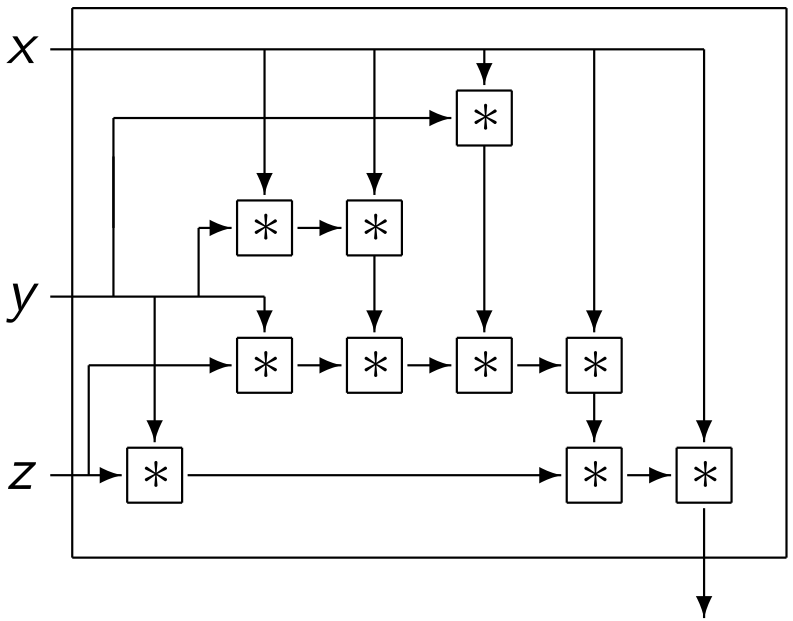


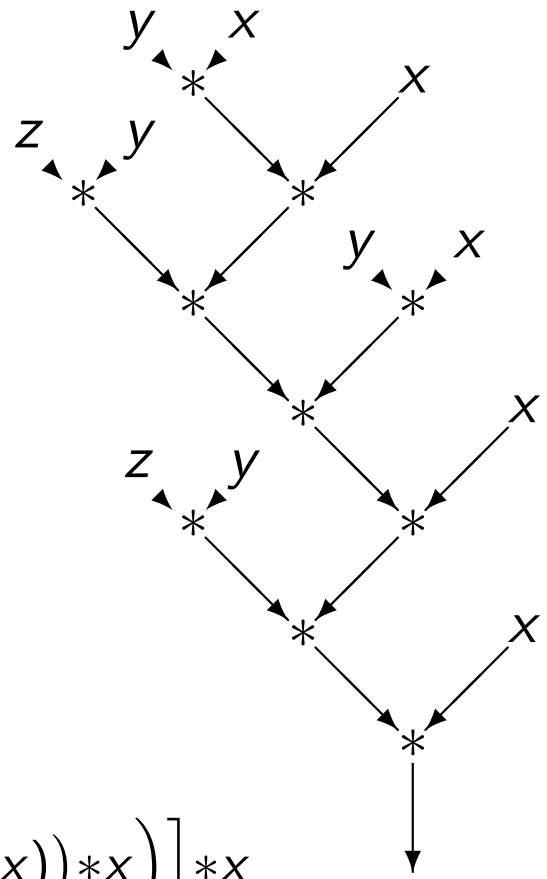
$$\Big[(z*y)*\Big(\Big(\big[(z*y)*((y*x)*x)\big]*(y*x)\Big)*x\Big)\Big]*x$$



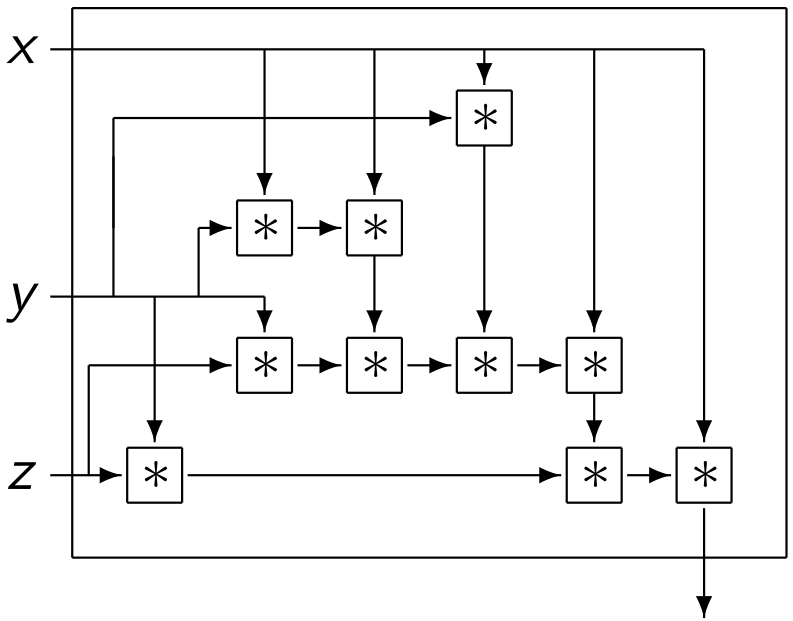


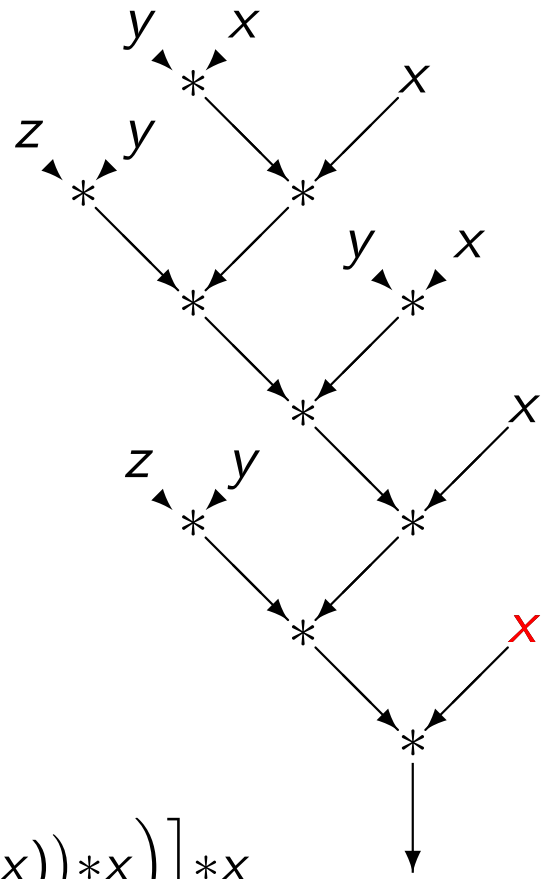
$$\Big[(z*y)*\Big(\Big(\big[(z*y)*((y*x)*x)\big]*(y*x)\Big)*x\Big)\Big]*x$$



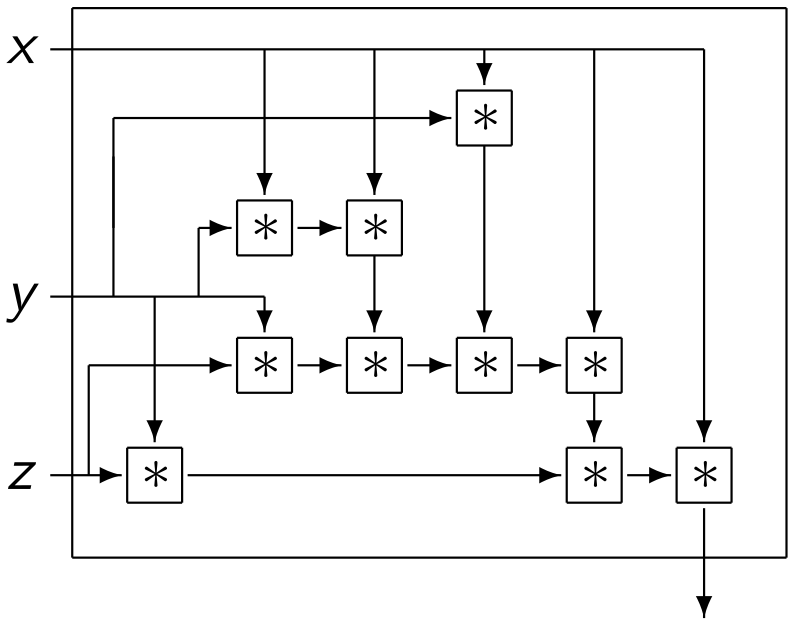


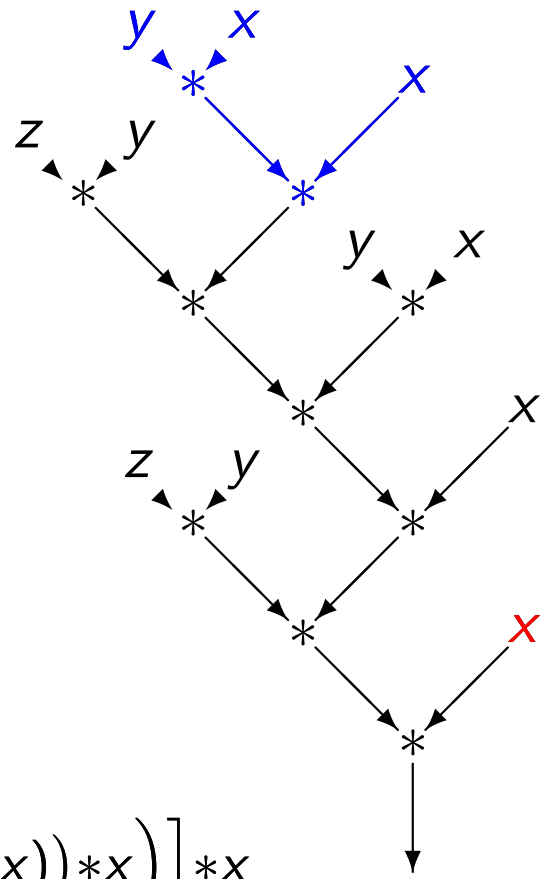
$$\Big[(z{*}y) * \Big(\big(\big[(z{*}y){*}((y{*}x){*}x)\big]{*}(y{*}x)\big){*}x\Big)\Big]{*}x$$



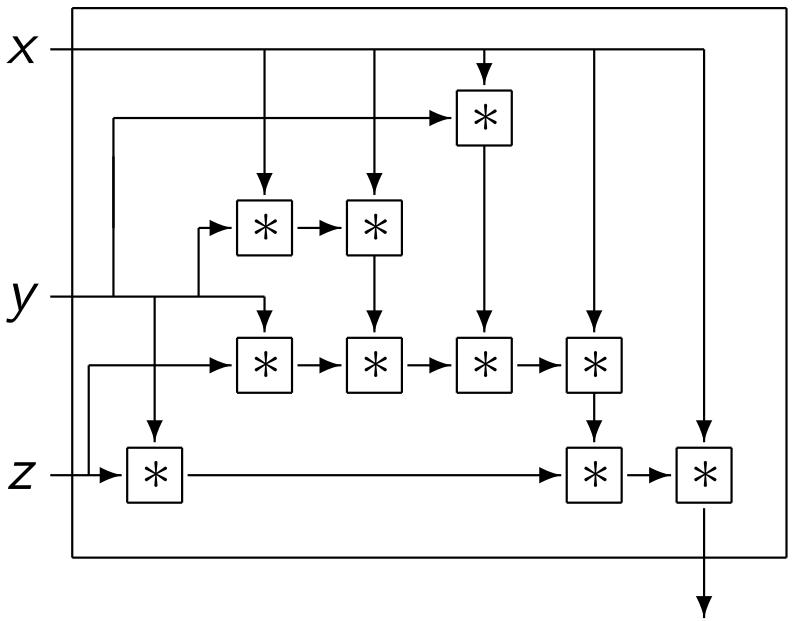


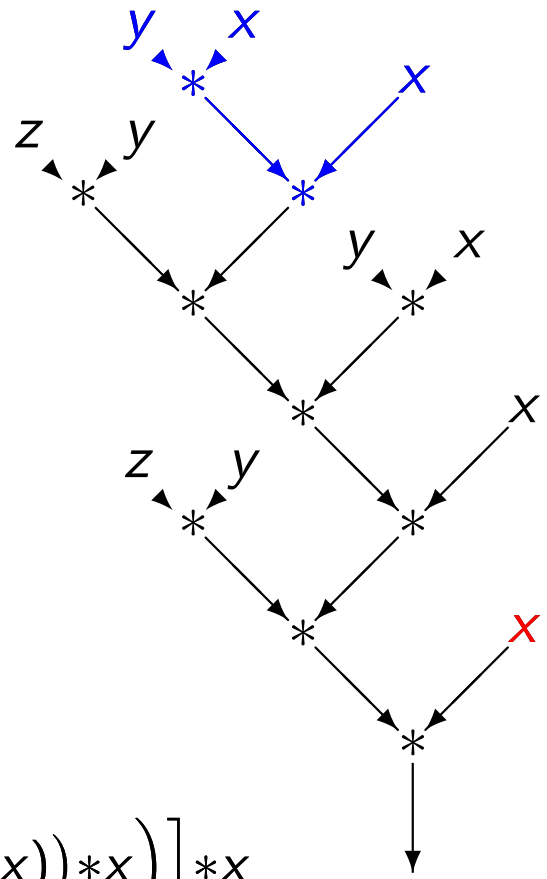
$$\Big[(z{*}y) * \Big(\big(\big[(z{*}y){*}((y{*}x){*}x)\big]{*}(y{*}x)\big){*}x\Big)\Big]{*}x$$





$$\Big[(z{*}y) * \Big(\big(\big[(z{*}y){*}\big((y{*}x){*}x\big)\big]{*}(y{*}x)\big){*}x\Big)\Big]{*}x$$





$$\Big[(z{*}y) * \Big(\big(\big[(z{*}y){*}\big((y{*}x){*}x\big)\big]{*}(y{*}x)\big){*}x\Big)\Big]{*}x$$

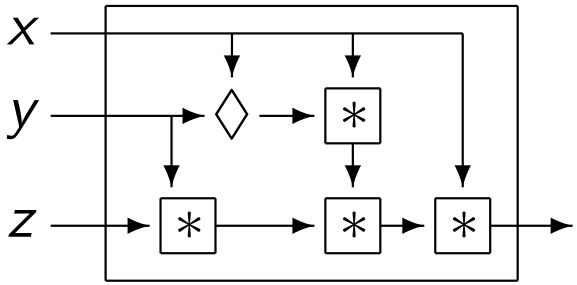
A **small** change in a circuit is a change high in its tree.

1. D. Clark, Evolution of algebraic terms 1: term to term operation continuity, *International Journal of Algebra and Computation*, Vol. 23, No. 5 (2013) 1175-1205.
2. D. Clark, M. Keijzer, L. Spector, Evolution of algebraic terms 2: the deep drilling algorithm, *International Journal of Algebra and Computation*, Vol. 26, No. 6 (2016) 1141- 1176.
3. D. Clark, L. Spector, Evolution of algebraic terms 3: beam algorithms and term continuity, *International Journal of Algebra and Computation*, Vol. 28, No. 05 (2018), 759-790.

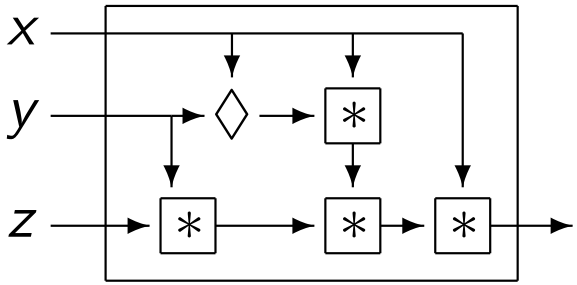
1. D. Clark, Evolution of algebraic terms 1: term to term operation continuity, *International Journal of Algebra and Computation*, Vol. 23, No. 5 (2013) 1175-1205.
2. D. Clark, M. Keijzer, L. Spector, Evolution of algebraic terms 2: the deep drilling algorithm, *International Journal of Algebra and Computation*, Vol. 26, No. 6 (2016) 1141- 1176.
3. D. Clark, L. Spector, Evolution of algebraic terms 3: beam algorithms and term continuity, *International Journal of Algebra and Computation*, Vol. 28, No. 05 (2018), 759-790.

Conjecture. *If a gate is term continuous, then the value computed at many nodes high enough in the circuit tree will have no effect on the output value.*

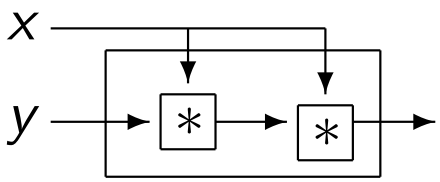
1. D. Clark, Evolution of algebraic terms 1: term to term operation continuity, *International Journal of Algebra and Computation*, Vol. 23, No. 5 (2013) 1175-1205.
2. D. Clark, M. Keijzer, L. Spector, Evolution of algebraic terms 2: the deep drilling algorithm, *International Journal of Algebra and Computation*, Vol. 26, No. 6 (2016) 1141- 1176.
3. D. Clark, L. Spector, Evolution of algebraic terms 3: beam algorithms and term continuity, *International Journal of Algebra and Computation*, Vol. 28, No. 05 (2018), 759-790.
4. D. Clark, N. Falco, Evolution of algebraic terms 4: a two-parent evolutionary algorithm, (under construction).



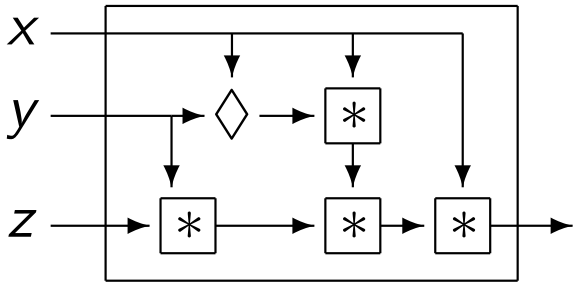
$$\big((z * y) * (\Diamond * x)\big) * x$$



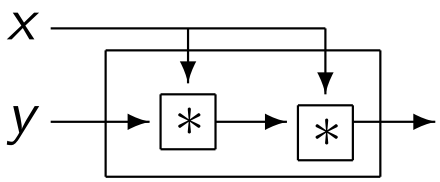
$$\big((z{*}y){*}(\Diamond{*}x)\big){*}x$$



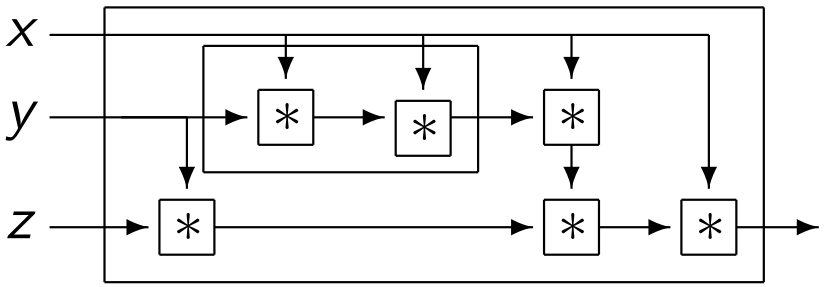
$$(y{*}x){*}x$$



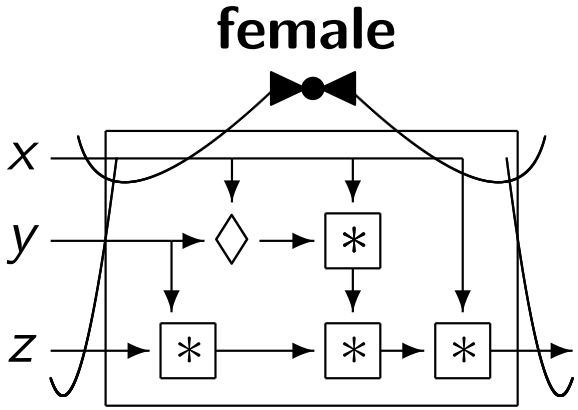
$$\big((z*y)*(\Diamond*x)\big)*x$$



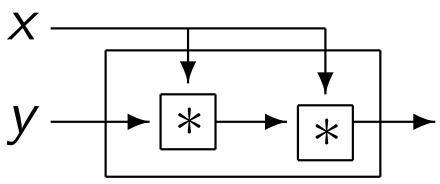
$$(y*x)*x$$



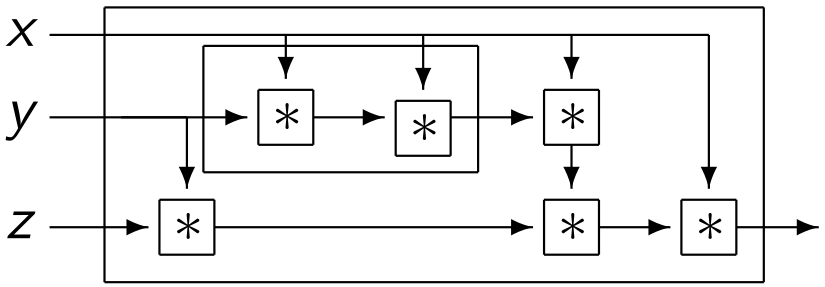
$$\big((z*y)*([(y*x)*x]*x)\big)*x$$



$$\big((z{*}y){*}(\Diamond{*}x)\big){*}x$$

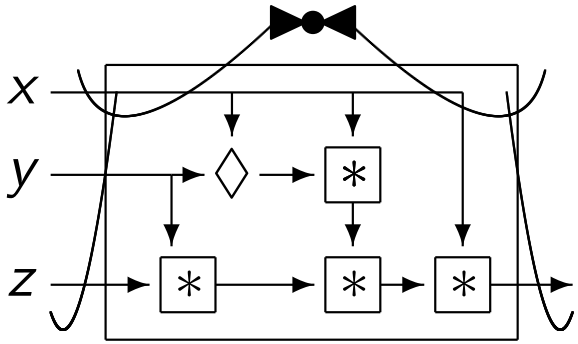


$$(y{*}x){*}x$$



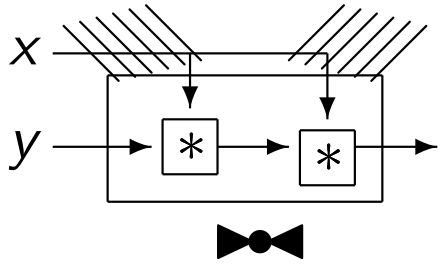
$$\big((z{*}y){*}([(y{*}x){*}x]{*}x)\big){*}x$$

female

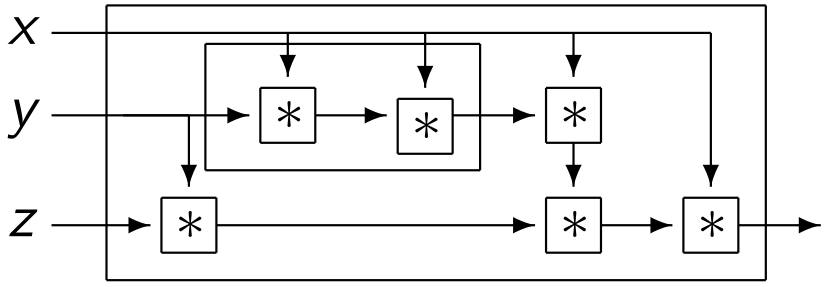


$$\big((z{*}y){*}(\Diamond{*}x)\big){*}x$$

male

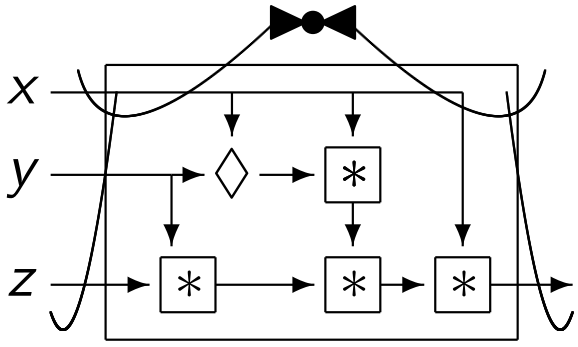


$$(y{*}x){*}x$$



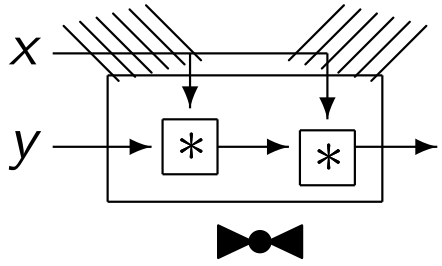
$$\big((z{*}y){*}([(y{*}x){*}x]{*}x)\big){*}x$$

female

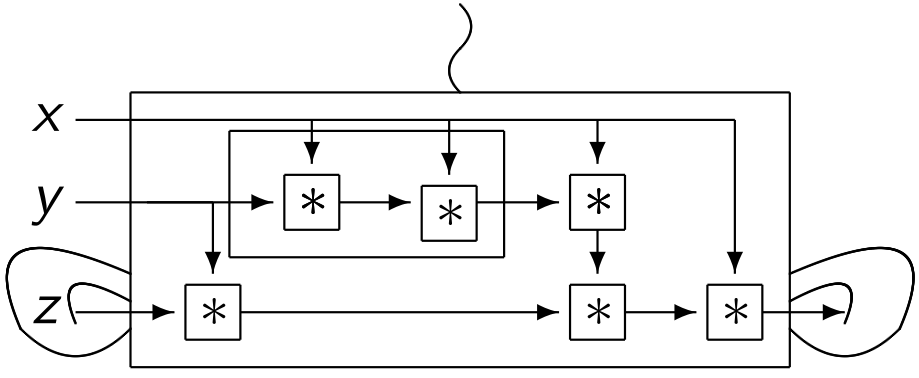


$$\big((z{*}y){*}(\Diamond{*}x)\big){*}x$$

male

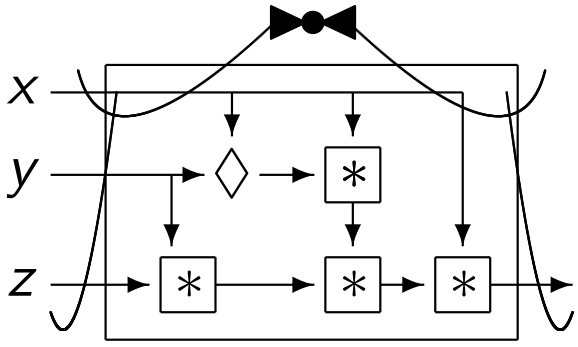


$$(y{*}x){*}x$$



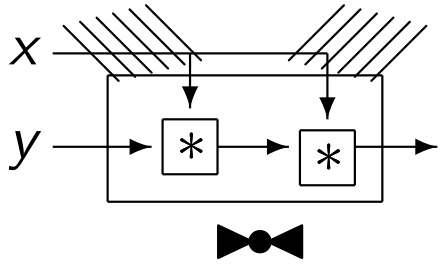
$$\big((z{*}y){*}([(y{*}x){*}x]{*}x)\big){*}x$$

female

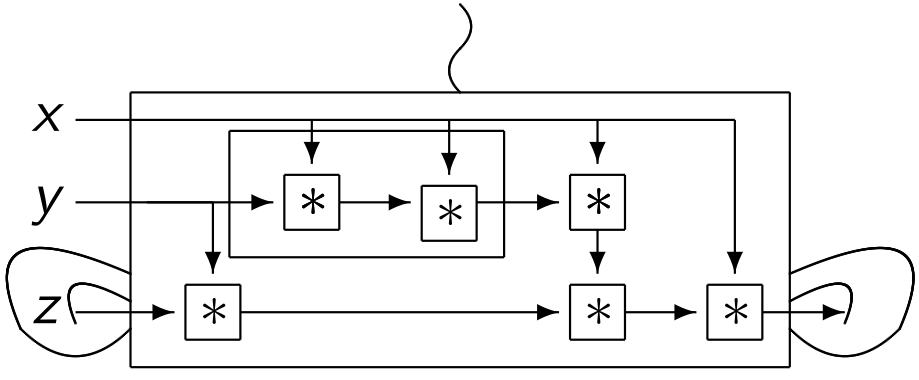


$$\big((z{*}y){*}(\Diamond{*}x)\big){*}x$$

male



$$(y{*}x){*}x$$



$$\big((z{*}y){*}([(y{*}x){*}x]{*}x)\big){*}x$$

son

female

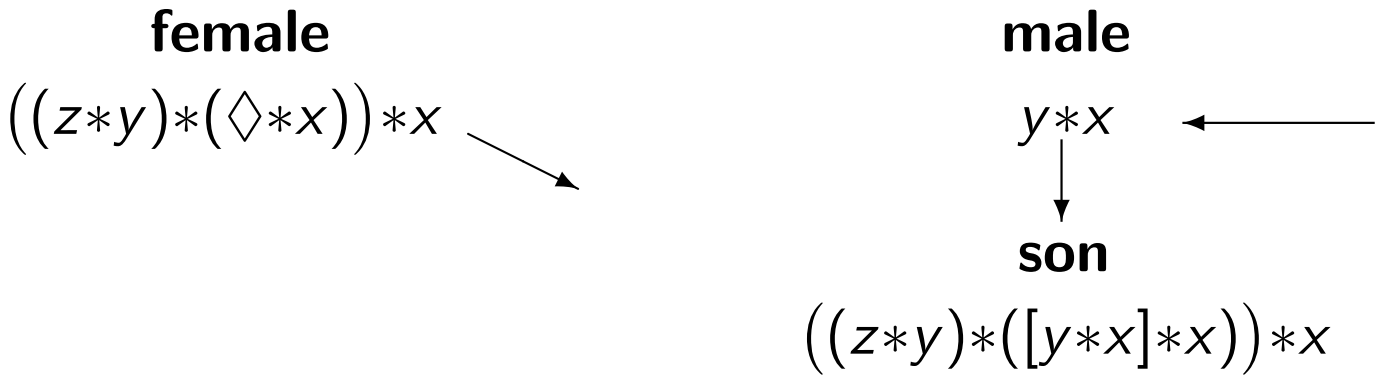
$$\big((z{*}y){*}(\Diamond{*}x)\big){*}x$$

female

$$\big((z{*}y){*}(\Diamond{*}x)\big){*}x$$

male

$$y{*}x \longleftarrow$$


female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$

female

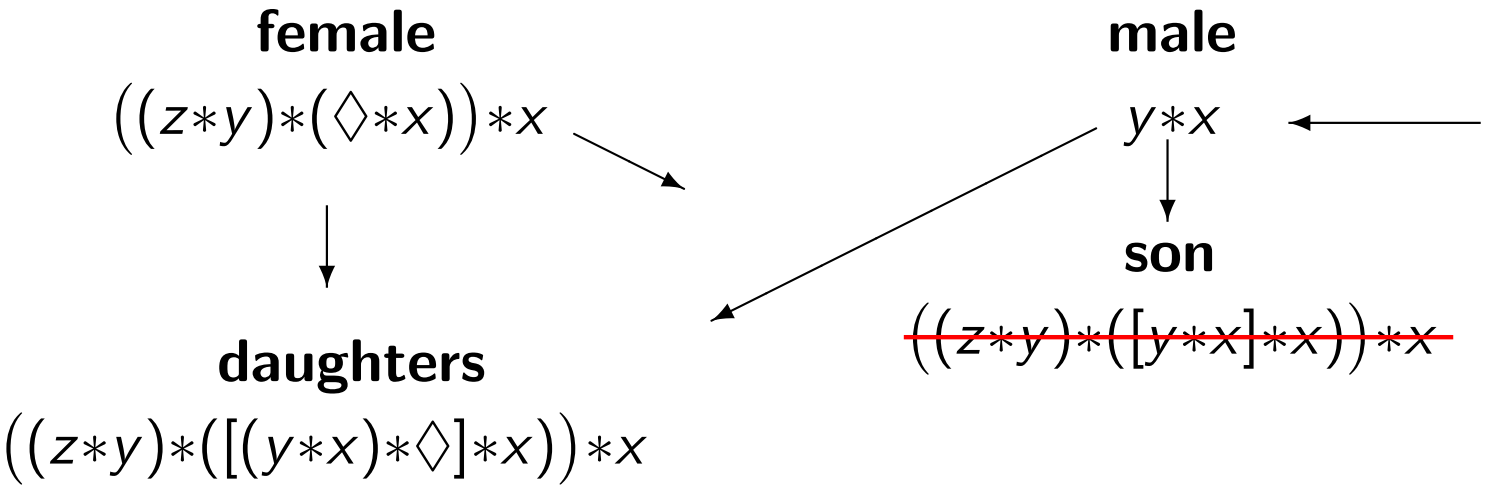
$$\big((z{*}y){*}(\Diamond{*}x)\big){*}x$$

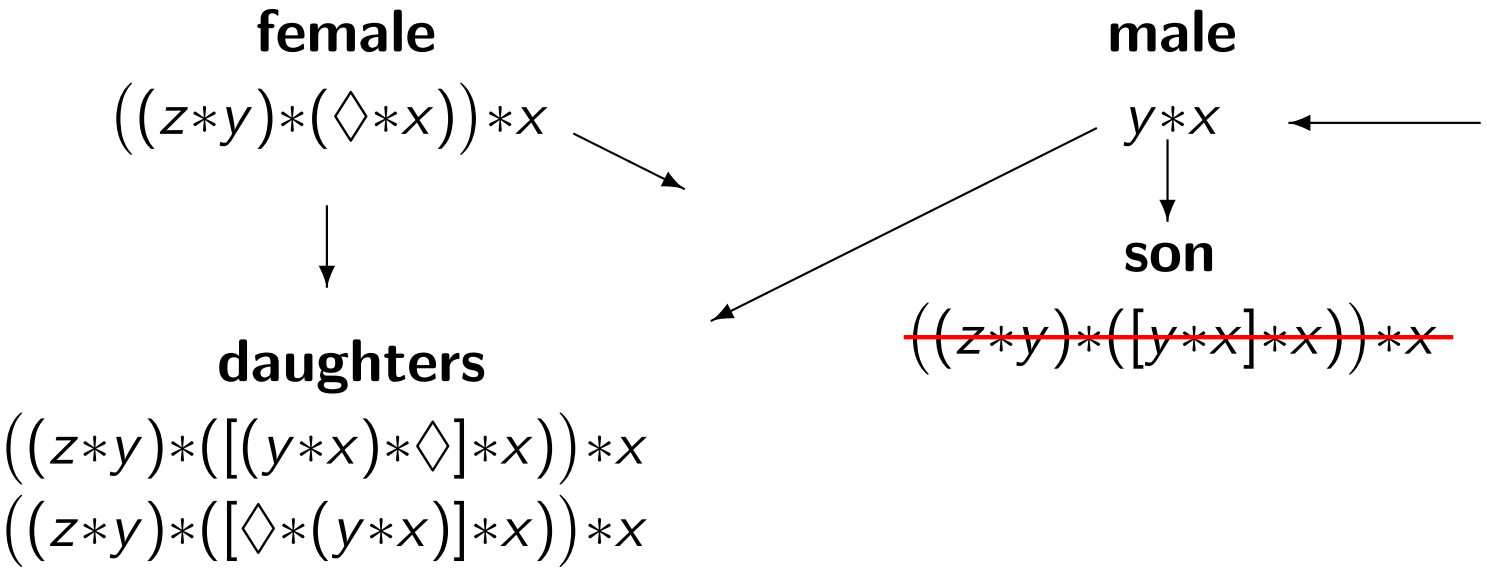
male

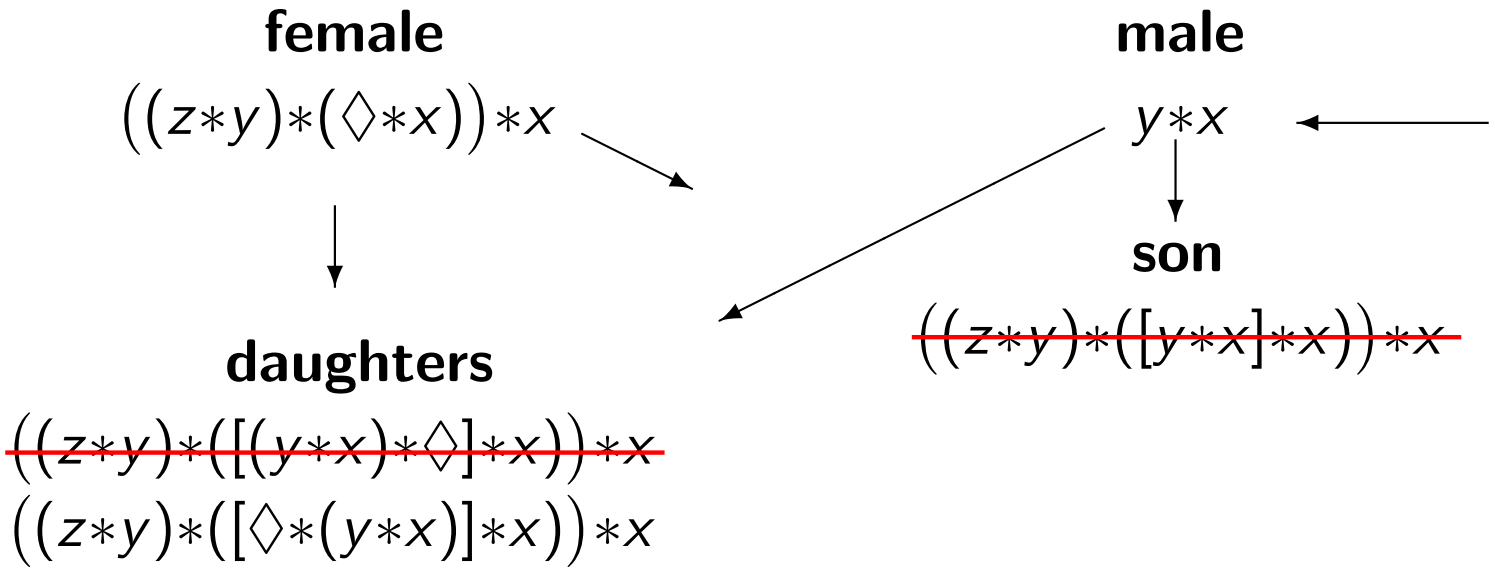
$$y{*}x$$

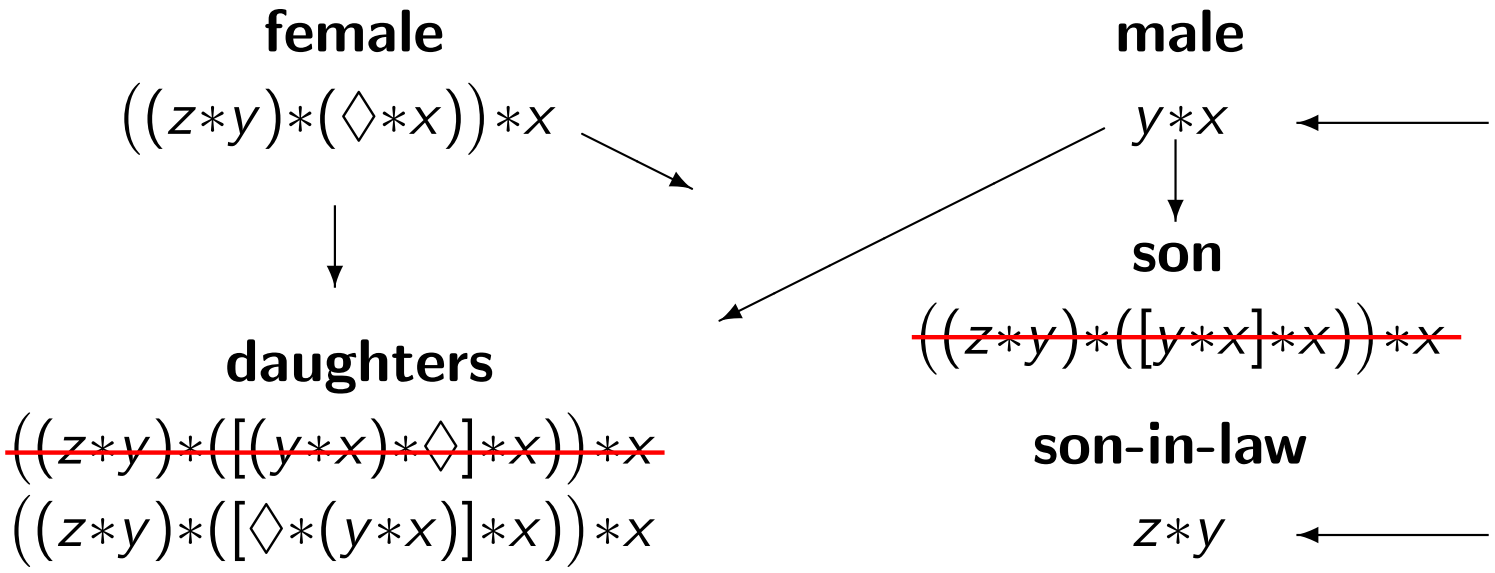
son

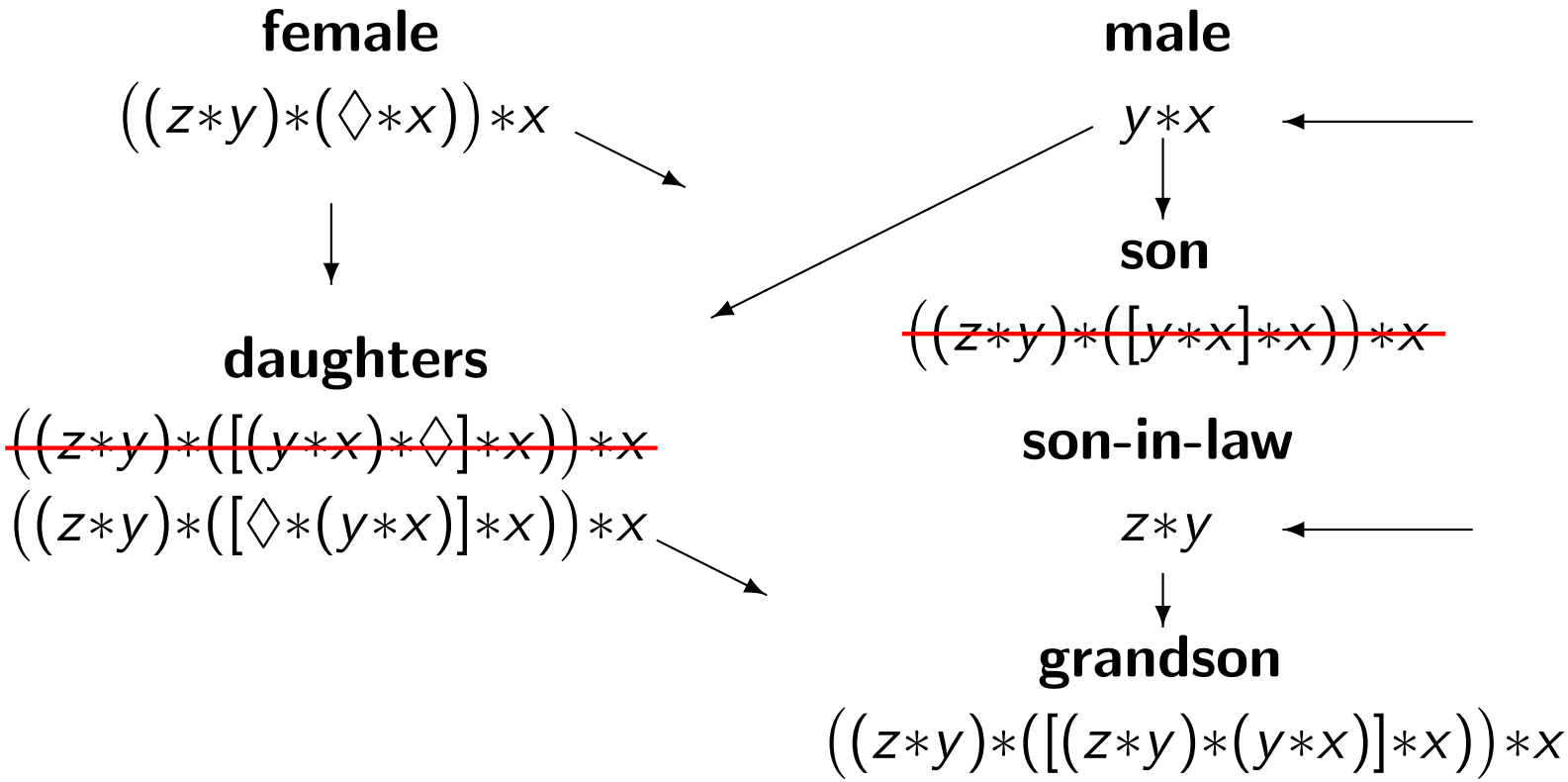
$$\cancel{\big((z{*}y){*}([y{*}x]{*}x)\big){*}x}$$

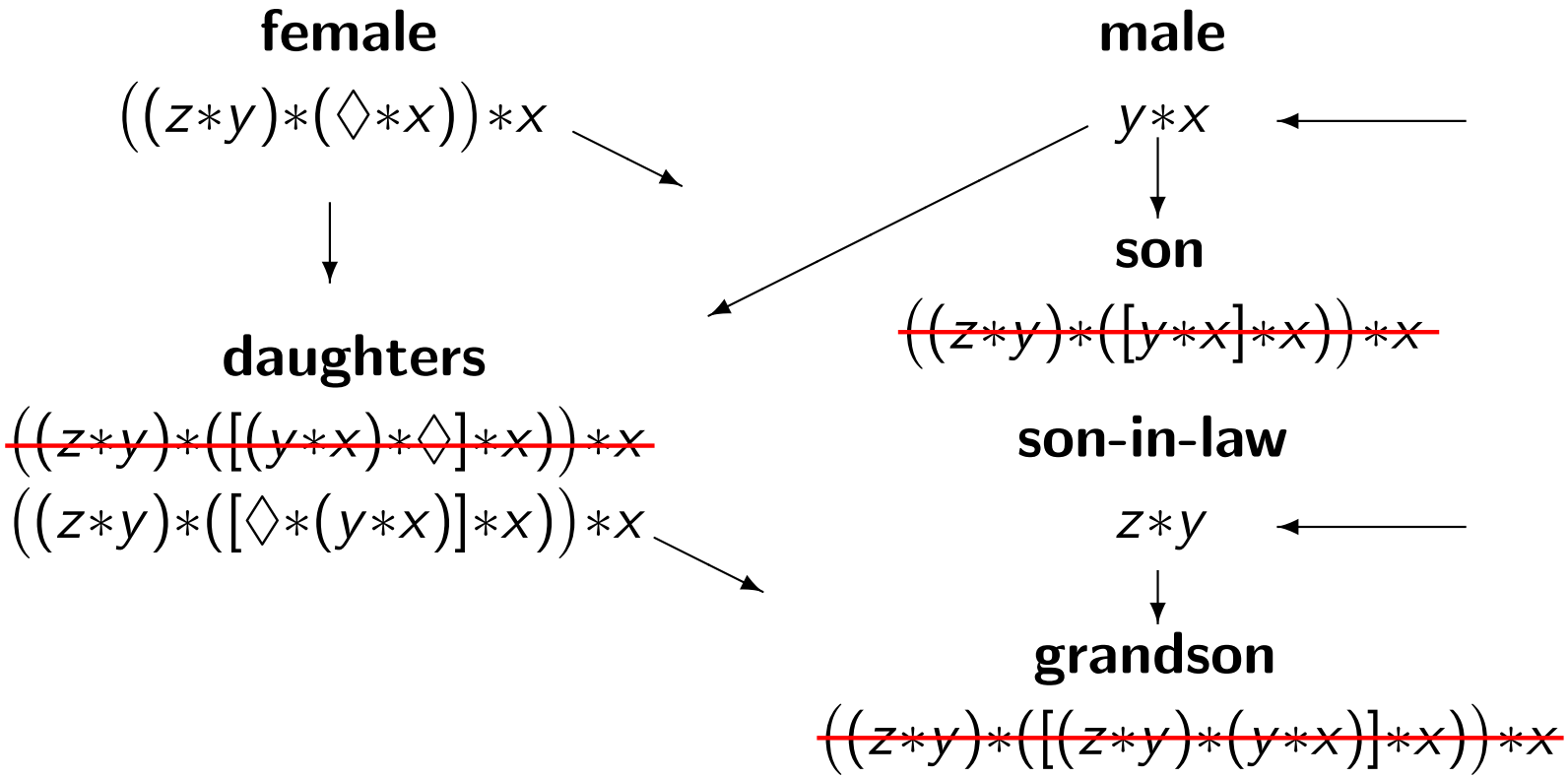

female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$

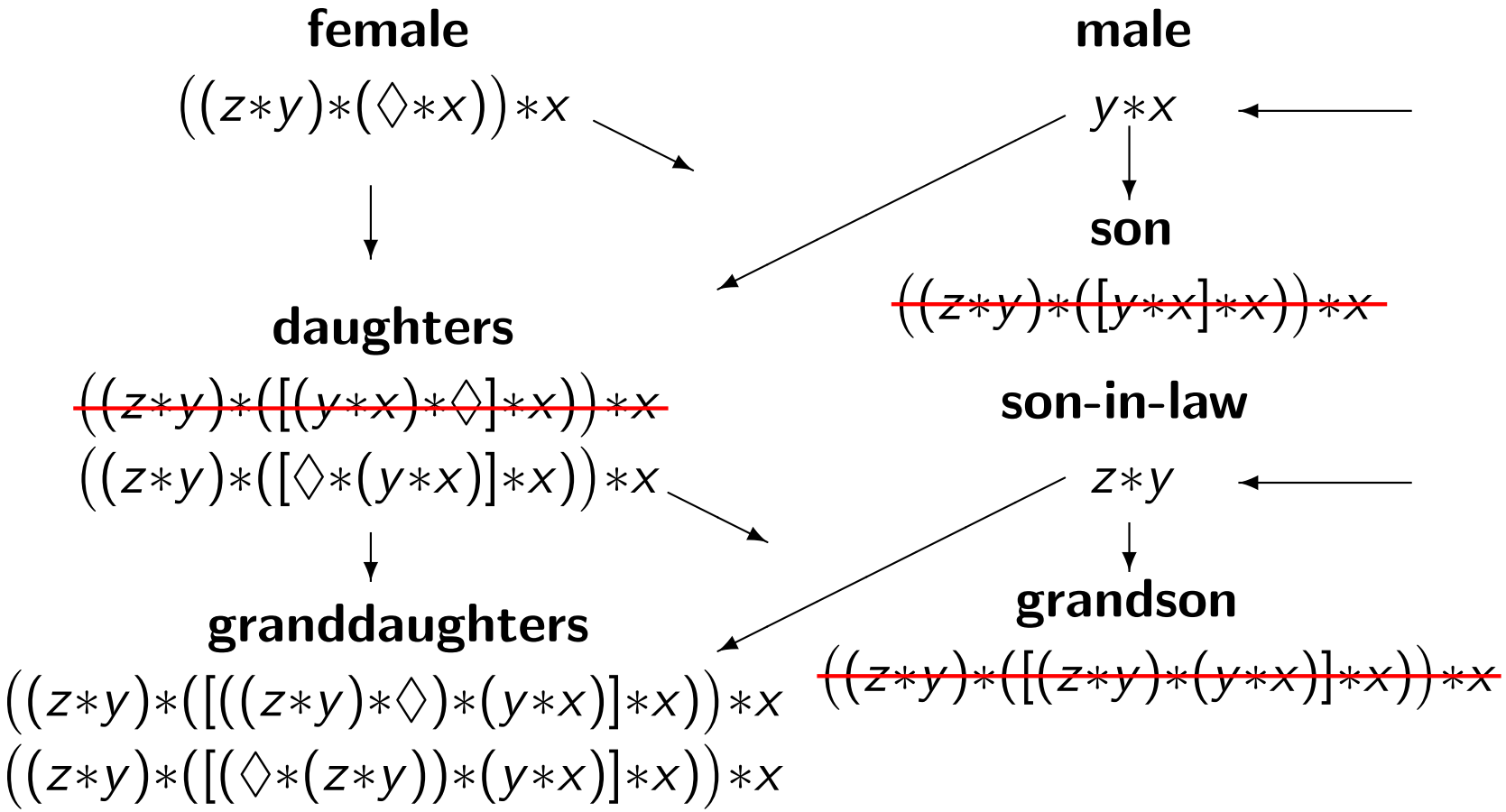

female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$
$((z*y)*([\Diamond*(y*x)]*x))*x$

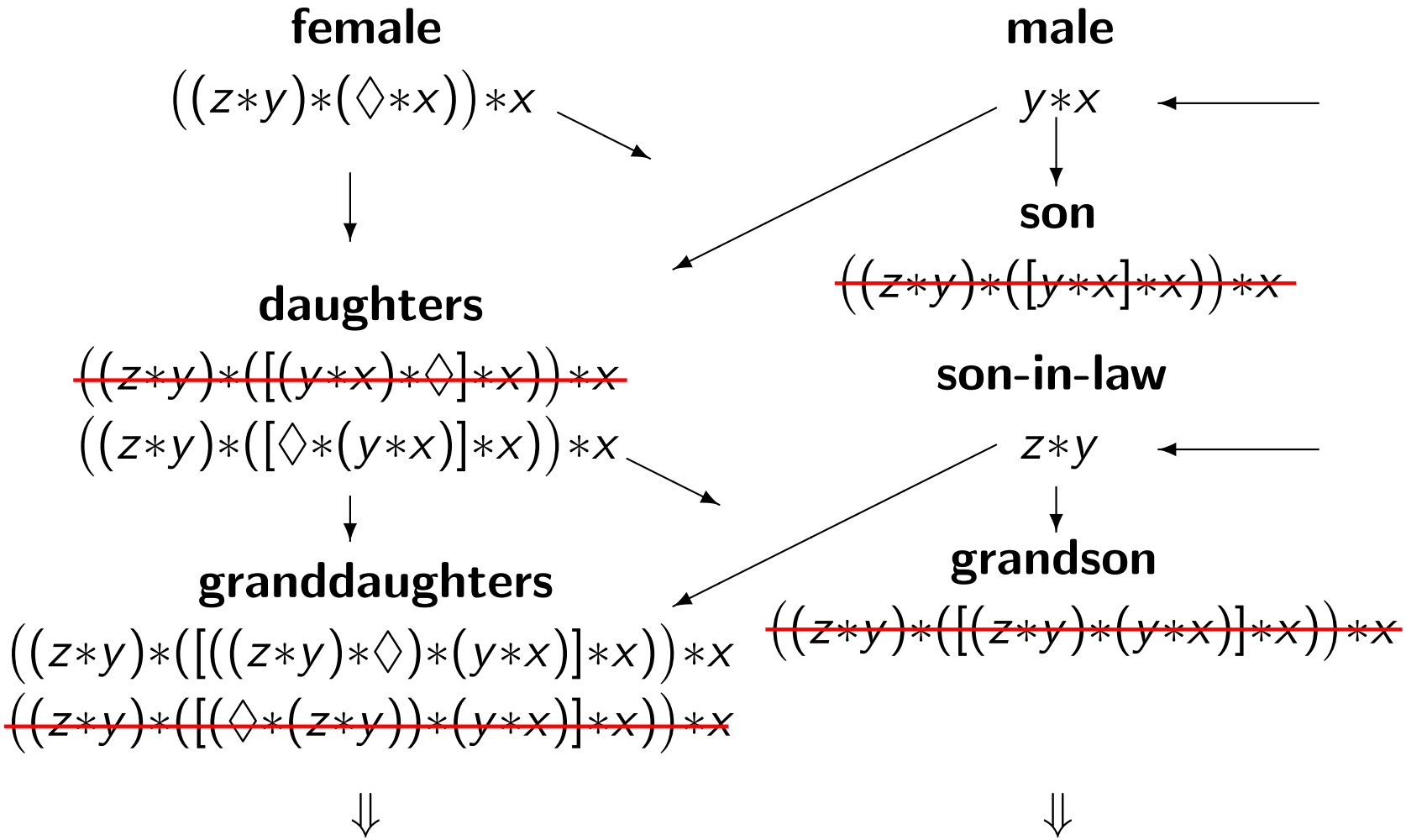

female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$
$((z*y)*([\Diamond*(y*x)]*x))*x$

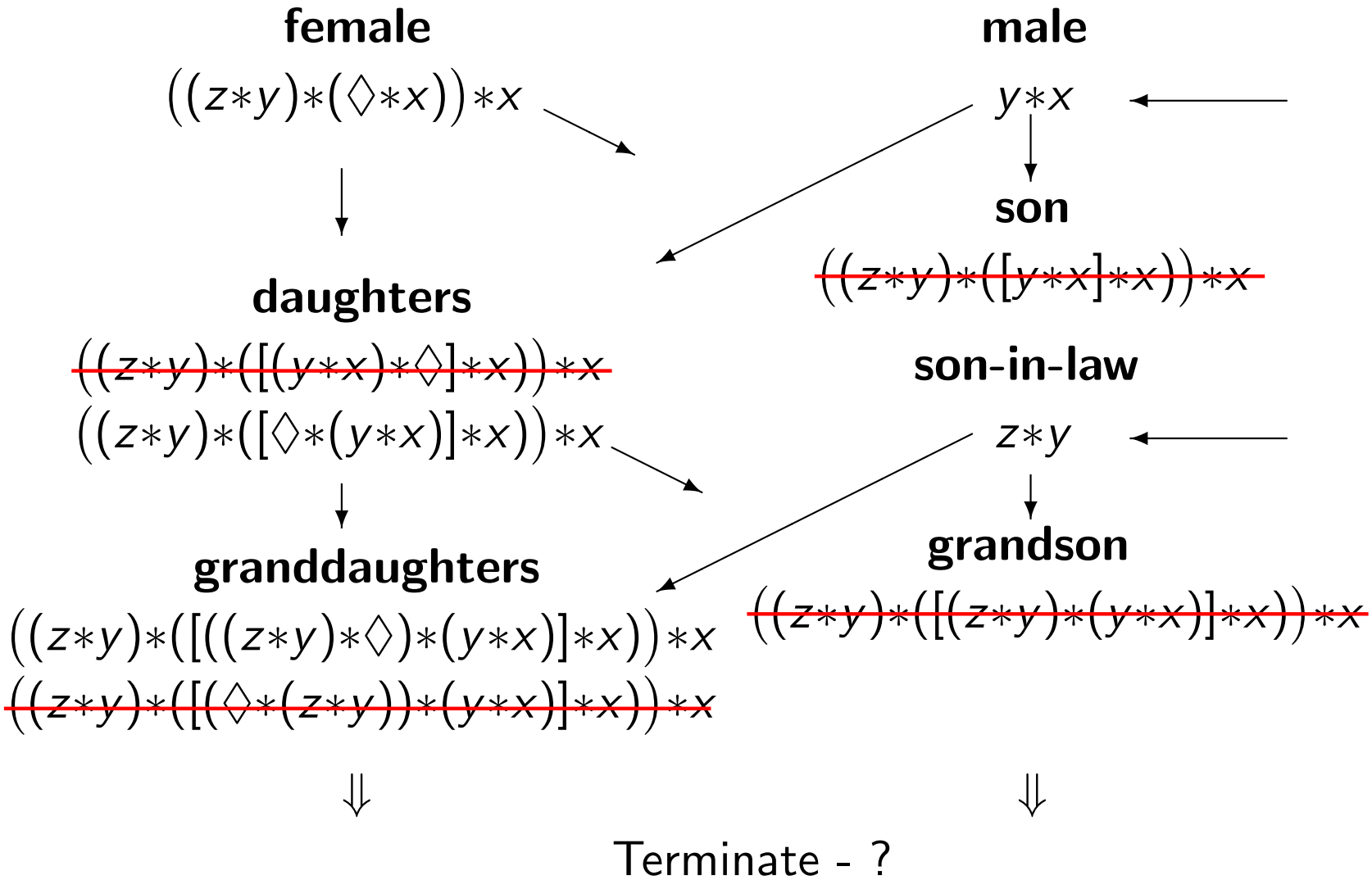

female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$
$((z*y)*([\Diamond*(y*x)]*x))*x$
son-in-law
$z*y$


female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$
$((z*y)*([\Diamond*(y*x)]*x))*x$
son-in-law
$z*y$
grandson
$((z*y)*([(z*y)*(y*x)]*x))*x$

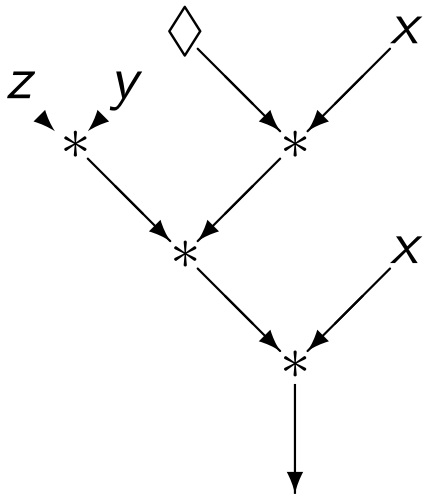

female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$
$((z*y)*([\Diamond*(y*x)]*x))*x$
son-in-law
$z*y$
grandson
$((z*y)*([(z*y)*(y*x)]*x))*x$


female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$
$((z*y)*([\Diamond*(y*x)]*x))*x$
son-in-law
$z*y$
granddaughters
$((z*y)*([((z*y)*\Diamond)*(y*x)]*x))*x$
$((z*y)*([(\Diamond*(z*y))*(y*x)]*x))*x$
grandson
$((z*y)*([(z*y)*(y*x)]*x))*x$

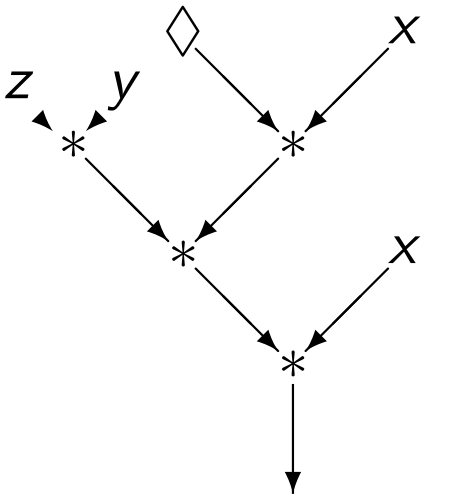

female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$
$((z*y)*([\Diamond*(y*x)]*x))*x$
son-in-law
$z*y$
granddaughters
$((z*y)*([((z*y)*\Diamond)*(y*x)]*x))*x$
$((z*y)*([(\Diamond*(z*y))*(y*x)]*x))*x$
grandson
$((z*y)*([(z*y)*(y*x)]*x))*x$


female
$((z*y)*(\Diamond*x))*x$
male
$y*x$
son
$((z*y)*([y*x]*x))*x$
daughters
$((z*y)*([(y*x)*\Diamond]*x))*x$
$((z*y)*([\Diamond*(y*x)]*x))*x$
son-in-law
$z*y$
granddaughters
$((z*y)*([((z*y)*\Diamond)*(y*x)]*x))*x$
$((z*y)*([(\Diamond*(z*y))*(y*x)]*x))*x$
grandson
$((z*y)*([(z*y)*(y*x)]*x))*x$
Terminate - ?

$$\Big(\big(z{*}y\big){*}\big(\Diamond{*}x\big)\Big){*}x$$

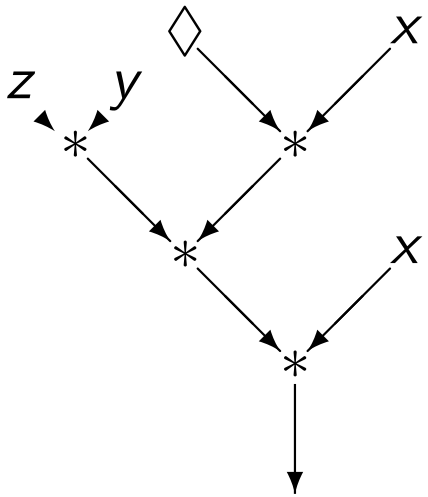


$$\Big(\big(z{*}y\big){*}\big(\Diamond{*}x\big)\Big){*}x$$



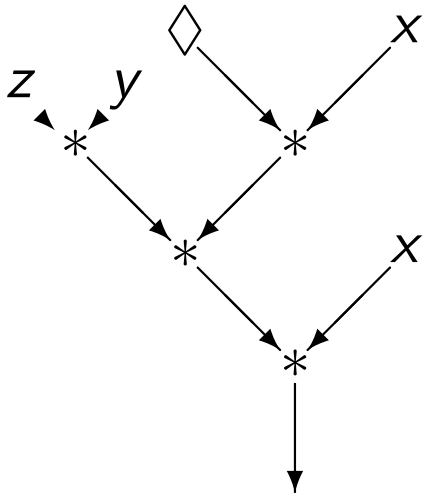
$$\Big(\big(z*y\big)*\big(\Diamond*x\big)\Big)*x$$

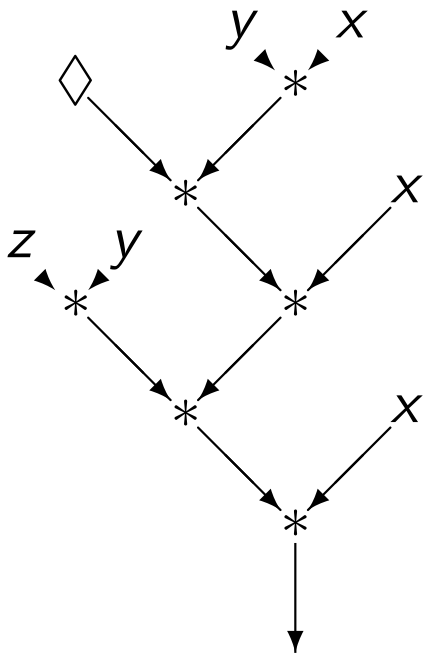
$$y*x$$



$$\Big((z*y)*(\Diamond*x)\Big)*x \qquad \Big((z*y)*\Big((\Diamond*(y*x))*x\Big)\Big)*x$$

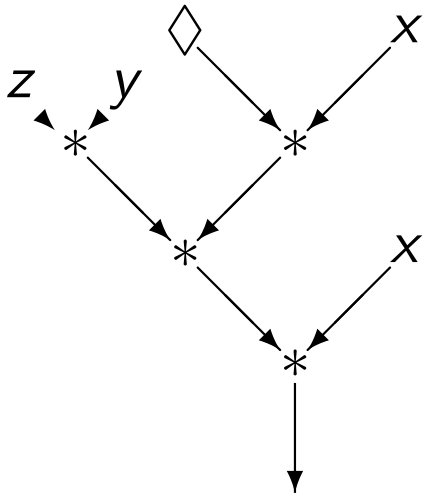
$y * x$





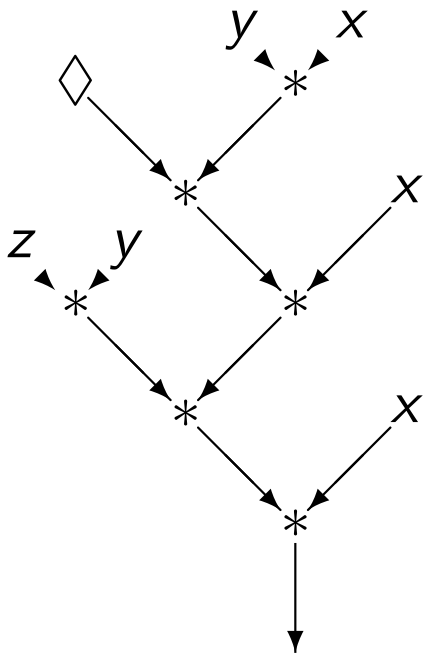
$$\Big((z*y)*(\Diamond*x)\Big)*x \qquad \Big((z*y)*\Big((\Diamond*(y*x))*x\Big)\Big)*x$$

$$y*x$$



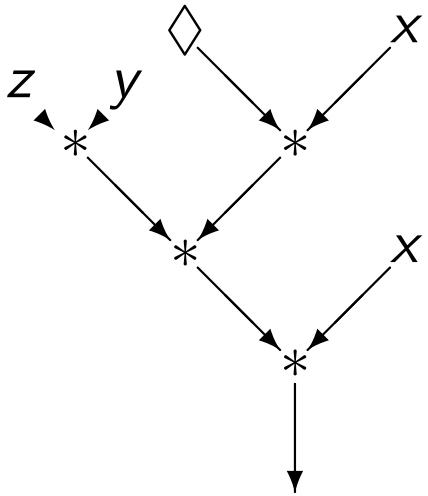
$$\Big((z*y)*(\Diamond*x)\Big)*x$$

$y * x$



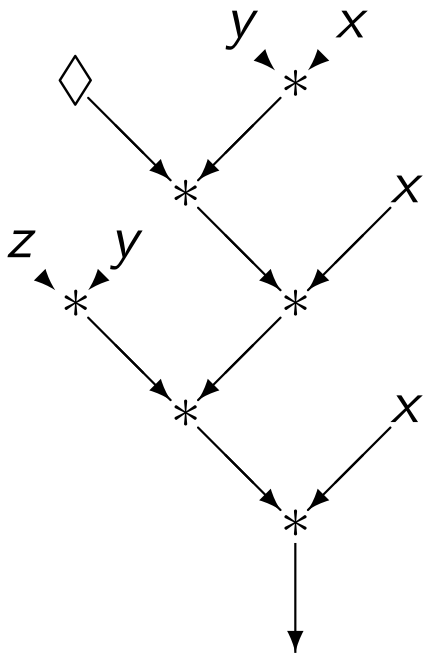
$$\Big((z*y)*\Big((\Diamond*(y*x))*x\Big)\Big)*x$$

$z * y$



$$\Big((z*y)*(\Diamond*x)\Big)*x$$

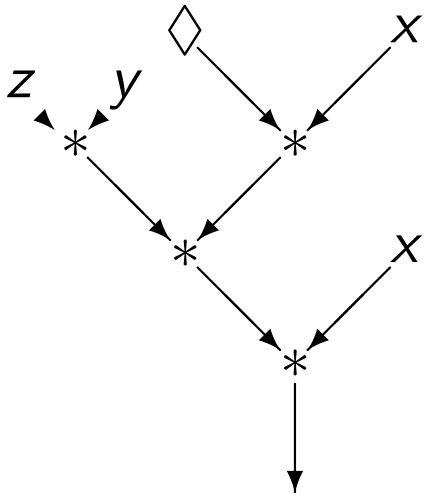
$$y * x$$



$$\Big((z*y)*\Big((\Diamond*(y*x))*x\Big)\Big)*x$$

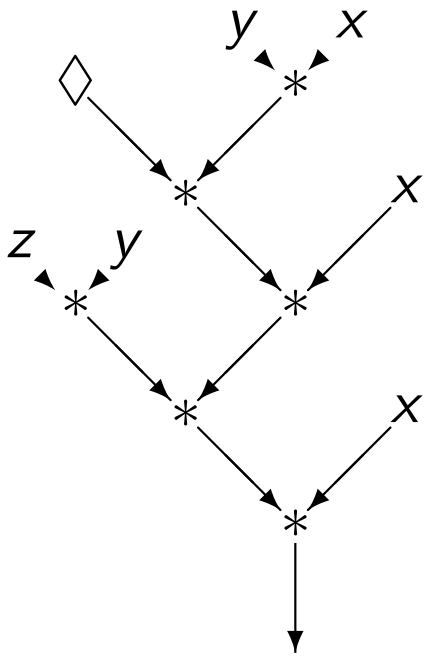
$$z * y$$

$$\Big((z*y)*\Big(\big(((z*y)*\Diamond)*(y*x)\big)*x\Big)\Big)*x$$



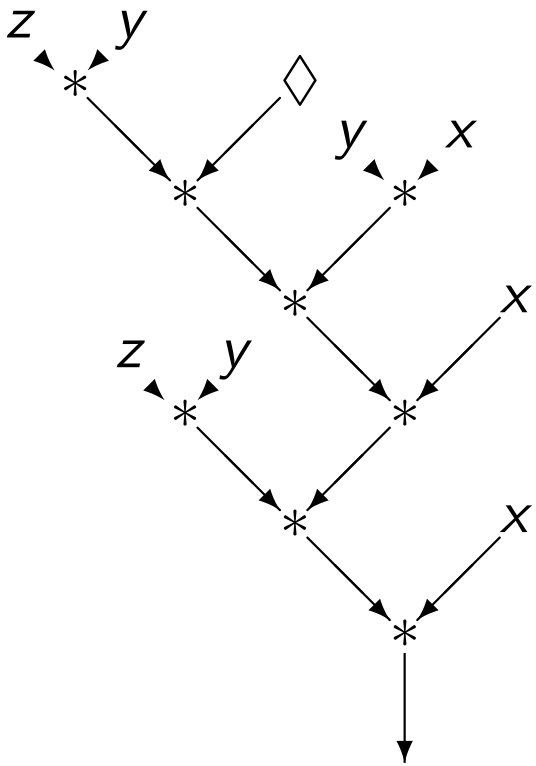
$$\Big((z{*}y){*}(\Diamond{*}x)\Big){*}x$$

$$y * x$$

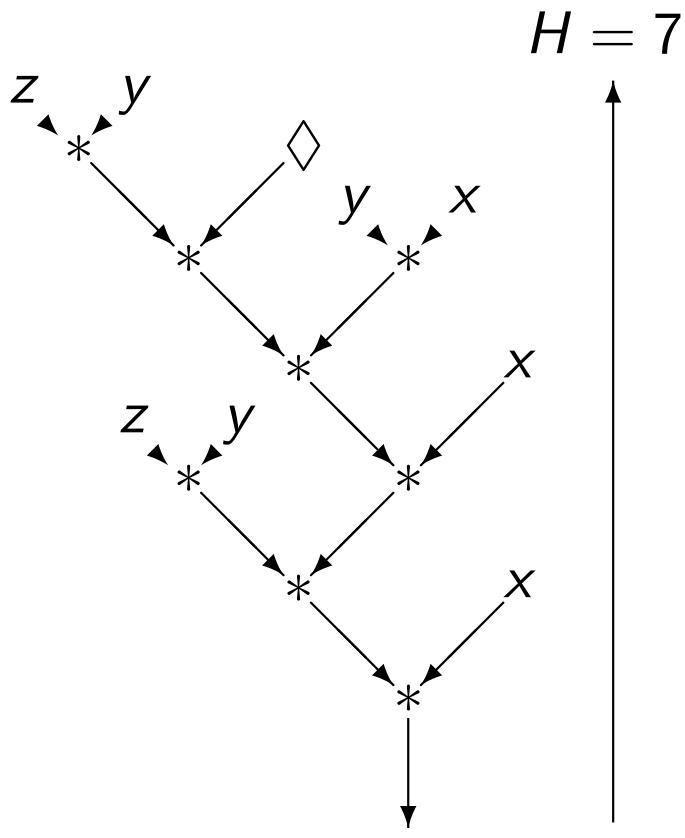


$$\Big((z{*}y){*}\Big((\Diamond{*}(y{*}x)){*}x\Big)\Big){*}x$$

$$z * y$$

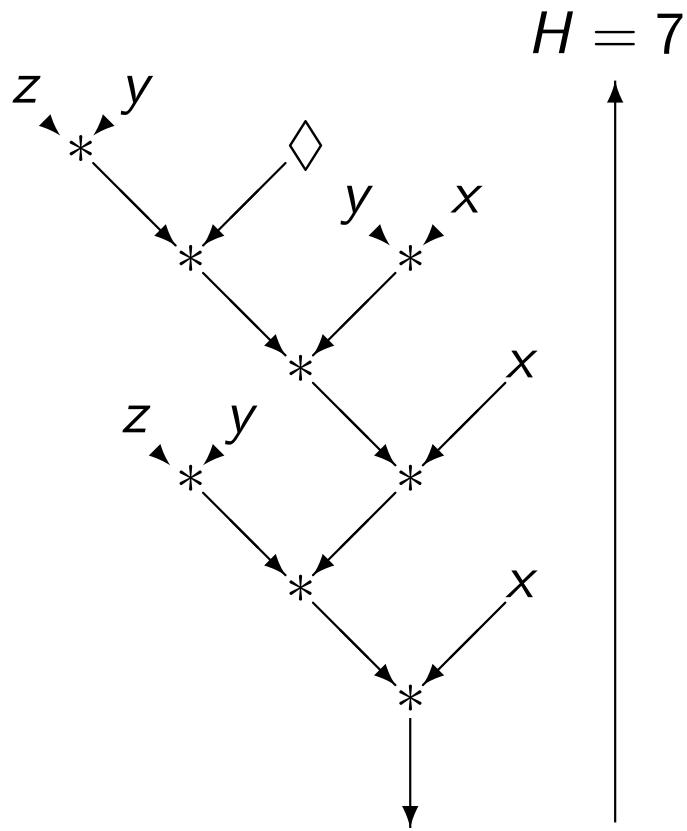


$$\Big((z{*}y){*}\Big(\big(((z{*}y){*}\Diamond){*}(y{*}x)\big){*}x\Big)\Big){*}x$$



$$\Big((z*y)*\Big(\big(\big((z*y)*\Diamond\big)*(y*x)\big)*x\Big)\Big)*x$$

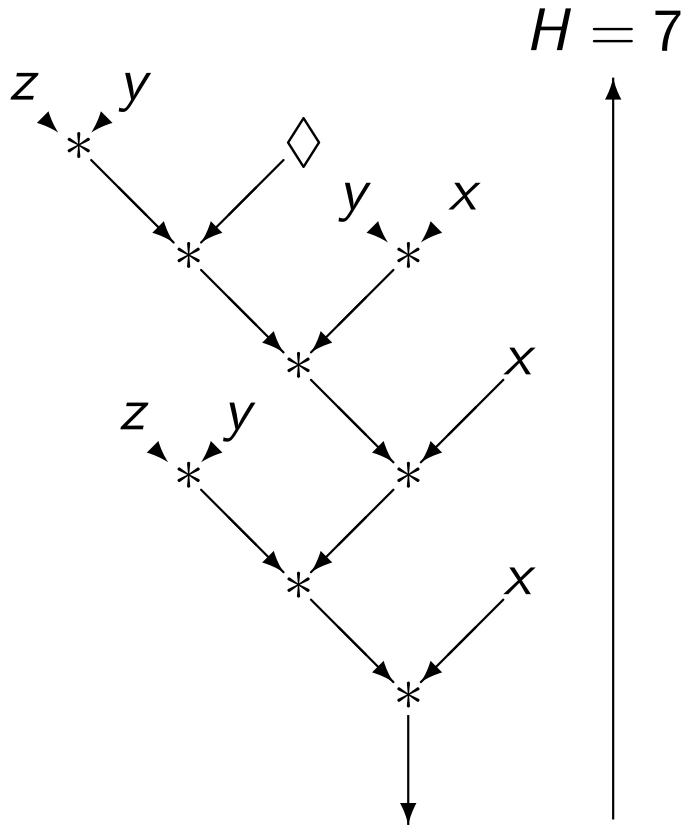
Theorem. *If* **G** *is a term continuous gate, then there is a number* λ *such that* $\frac{2}{3} \leq \lambda \leq 1$ *and*



$$\Big((z{*}y){*}\Big(\big(\big((z{*}y){*}\Diamond\big){*}(y{*}x)\big){*}x\Big)\Big){*}x$$

Theorem. *If* **G** *is a term continuous gate, then there is a number* λ *such that* $\frac{2}{3} \leq \lambda \leq 1$ *and*

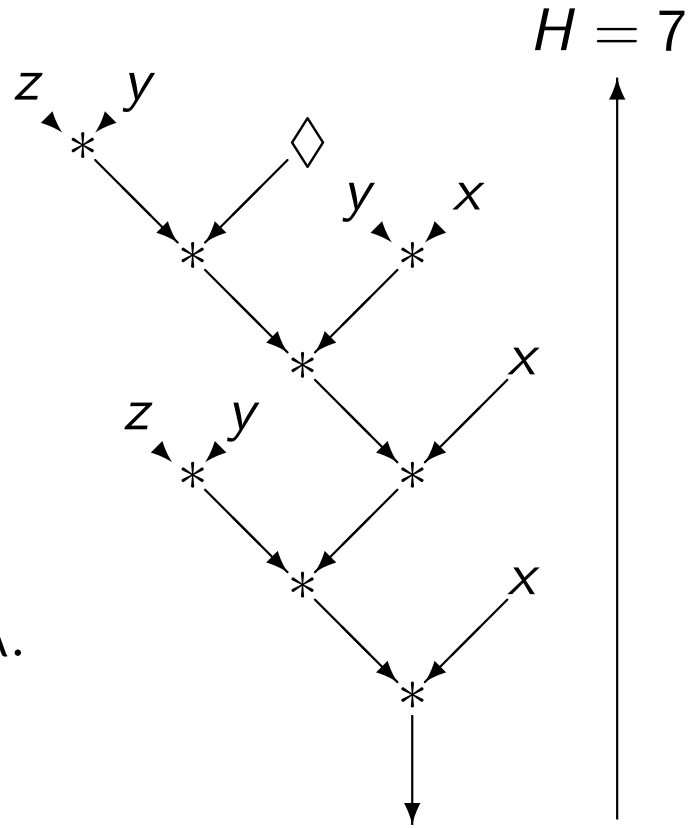
$$\lim_{H \to \infty} \frac{\#\Diamond\text{−dead circuits of height } H}{\#\Diamond\text{−circuits of height } H}$$



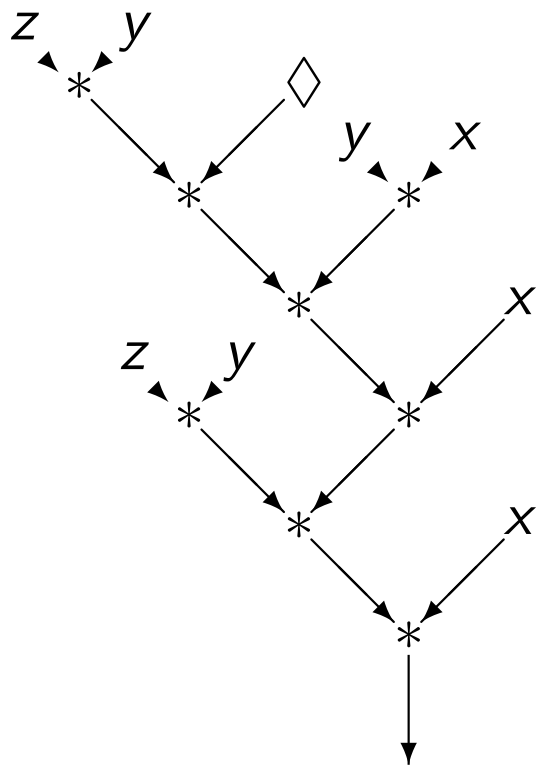
$$\Big((z{*}y){*}\Big(\big(\big((z{*}y){*}\Diamond\big){*}(y{*}x)\big){*}x\Big)\Big){*}x$$

Theorem. *If* **G** *is a term continuous gate, then there is a number* λ *such that* $\frac{2}{3} \leq \lambda \leq 1$ *and*

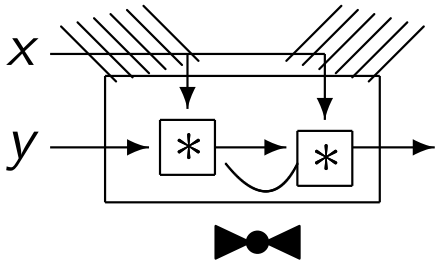
$$\lim_{H \to \infty} \frac{\#\Diamond\text{−dead circuits of height } H}{\#\Diamond\text{−circuits of height } H} = \lambda.$$



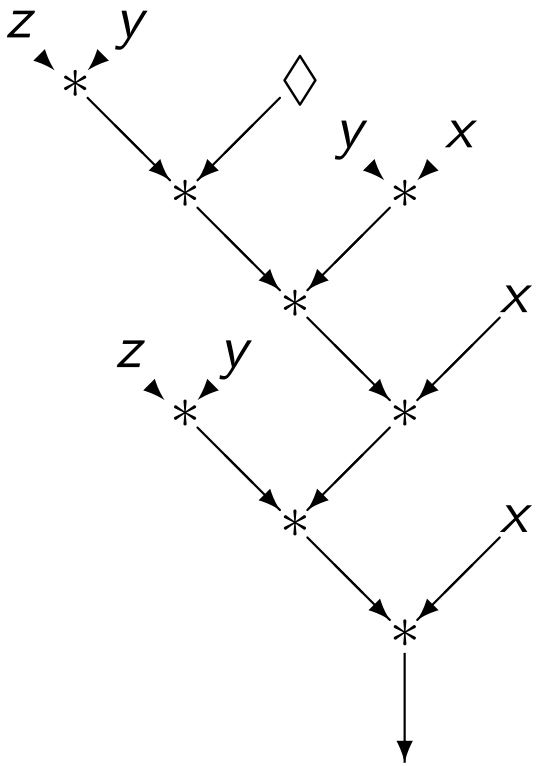
$$\Big((z{*}y){*}\Big(\big(\big((z{*}y){*}\Diamond\big){*}(y{*}x)\big){*}x\Big)\Big){*}x$$



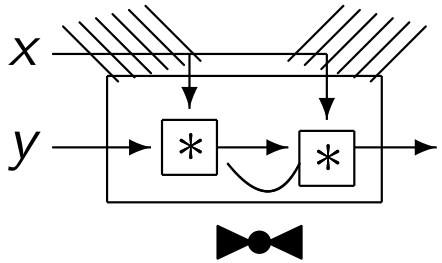
$$\Big((z*y)*\Big(\big(\big((z*y)*\Diamond\big)*(y*x)\big)*x\Big)\Big)*x$$



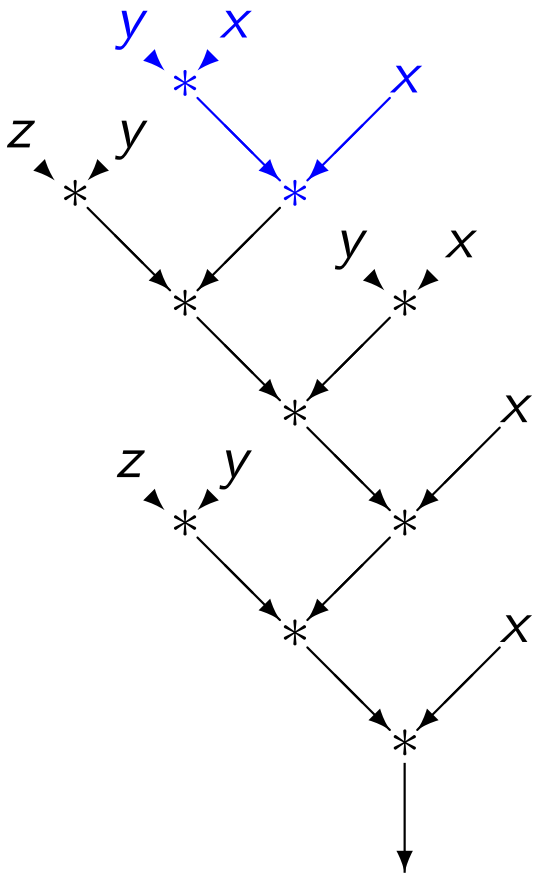
$$(y*x)*x$$



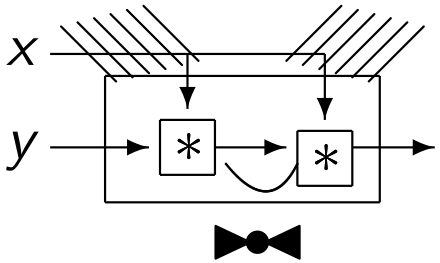
$$\Big((z*y)*\Big(\big(\big((z*y)*\Diamond\big)*(y*x)\big)*x\Big)\Big)*x$$



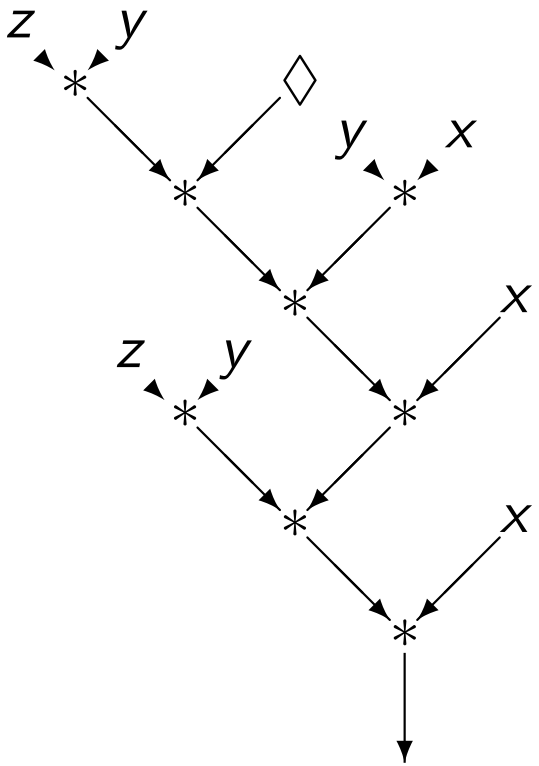
$$(y*x)*x$$



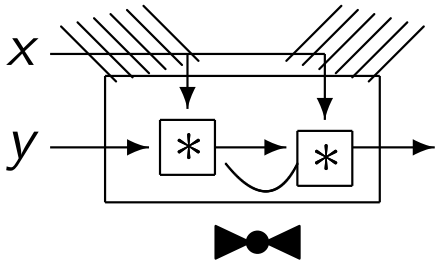
$$\Big((z*y)*\Big(\big(\big(((z*y)*((y*x)*x))*(y*x)\big)*x\big)\Big)*x$$



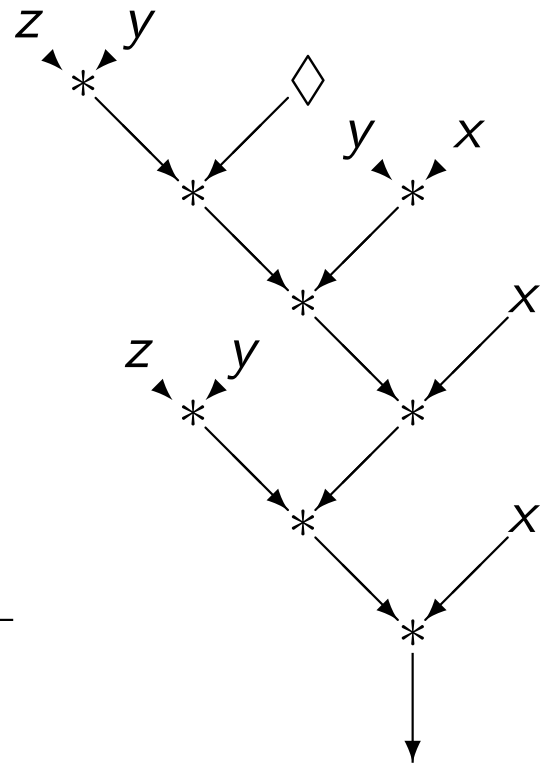
$$(y*x)*x$$

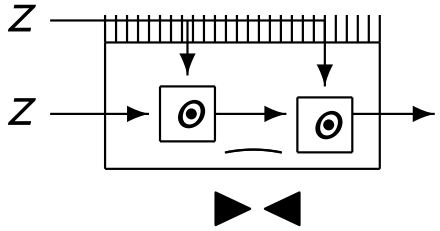


$$\Big((z*y)*\Big(\big(\big((z*y)*\Diamond\big)\big)*(y*x)\big)*x\Big)\Big)*x$$



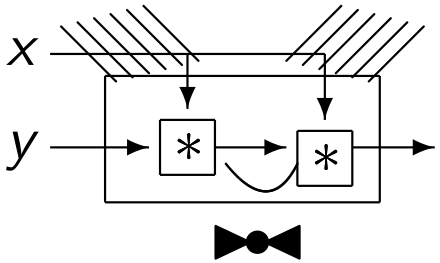
$(y*x)*x$



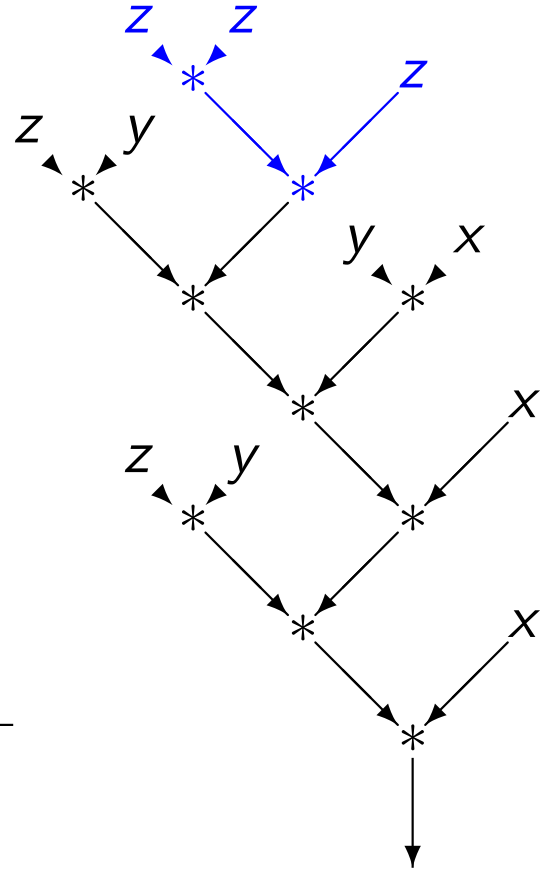


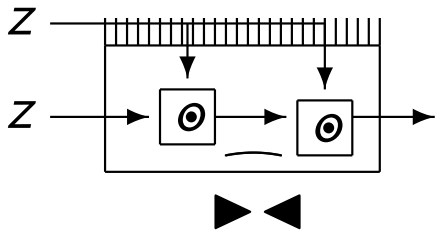
$(z*z)*z$

$$\Big((z*y)*\Big(\big(\big((z*y)*\Diamond\big)\big)*(y*x)\big)*x\Big)\Big)*x$$



$(y*x)*x$





$(z*z)*z$

$$\Big((z*y)*\Big(\big(\big((z*y)*\big((z*z)*z\big)\big)*(y*x)\big)*x\Big)\Big)*x$$

3-Element Gate

Possible target sequences: $3^{27} \approx 10^{13}$.

Typical solution to design problem, generated by **random search:**

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

3-Element Gate

Possible target sequences: $3^{27} \approx 10^{13}$.

Typical solution to design problem, generated by **random search: 4 months**

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

3-Element Gate

Possible target sequences: $3^{27} \approx 10^{13}$.

Typical solution to design problem, generated

- by **random search: 4 months**
- by **1-parent evolution: 5 minutes**

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

3-Element Gate

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Possible target sequences: $3^{27} \approx 10^{13}$.

Typical solution to design problem, generated

- by **random search: 4 months**
- by **1-parent evolution: 5 minutes**
- by **2-parent evolution:**

3-Element Gate

$*$	0	1	2
0	2	1	2
1	1	0	0
2	0	0	1

Possible target sequences: $3^{27} \approx 10^{13}$.

Typical solution to design problem, generated

- by **random search: 4 months**
- by **1-parent evolution: 5 minutes**
- by **2-parent evolution: 0.37 seconds**

$$(((((((((((((((((((((((((z*x)*((((((((z*(x*x))*y)*(x*(z*z)))*(((((y*x)*z)*y)*x))*(z*y))*(((z*y)*z)*y))*((z*x)*y))*(y*z))*(z*x)))*(z*x))*(y*(x*(x*y))))*((y*x)*((z*y)*y)))*(((z*z)*(z*z))*x))*(y*(x*x)))*(y*y))*(z*y))*(x*(((y*z)*y)*(x*z))))*((z*(z*x))*x))*((z*z)*x))*((z*y)*z))*((z*z)*y))*((((y*(z*(((y*((y*(y*z))*z))*x)*z)))*x)*(y*z))*x))*((x*(x*(((y*(z*y))*z)*((z*((y*(x*z))*z))*y))))*x))*((z*((((x*y)*x)*z)*(y*z)))*y))*((z*(y*z))*x))*(((y*z)*x)*(x*x)))*(((y*((x*(z*y))*x))*z)*x))*((x*z)*x))*((x*(z*y))*(x*y)))*((((y*z)*z)*z)*(x*x)))*((x*y)*(y*x)))*((x*z)*(z*x)))$$

5-Element Gate

$*$	0	1	2	3	4
0	3	2	0	4	0
1	2	4	1	1	1
2	1	1	3	0	1
3	1	3	4	1	0
4	0	1	0	1	0

Possible target sequences: $5^{125} \approx 10^{87}$.

5-Element Gate

$*$	0	1	2	3	4
0	3	2	0	4	0
1	2	4	1	1	1
2	1	1	3	0	1
3	1	3	4	1	0
4	0	1	0	1	0

Possible target sequences: $5^{125} \approx 10^{87}$.

Typical solution to design problem, generated by **random search: 10^{74} years**

5-Element Gate

$*$	0	1	2	3	4
0	3	2	0	4	0
1	2	4	1	1	1
2	1	1	3	0	1
3	1	3	4	1	0
4	0	1	0	1	0

Possible target sequences: $5^{125} \approx 10^{87}$.

Typical solution to design problem, generated

- by **random search: 10^{74} years**
- by **2-parent evolution:**

5-Element Gate

$*$	0	1	2	3	4
0	3	2	0	4	0
1	2	4	1	1	1
2	1	1	3	0	1
3	1	3	4	1	0
4	0	1	0	1	0

Possible target sequences: $5^{125} \approx 10^{87}$.

Typical solution to design problem, generated

- by **random search: 10^{74} years**
- by **2-parent evolution: 5.347 sec:**

$$(((((x*y)*(x*z))*((z*(y*(z*y)))*(((z*(y*((z*y)*x)))*((((((((((z*z)*x)*z)*((((((x*x)*y)*z)*(((z*(z*z))*z)*((x*z)*(((x*(z*z))*x)*((y*(z*(y*x)))*((z*(x*x))*((((x*y)*z)*((y*(x*(x*y)))*((x*((z*z)*x))*((y*z)*((((z*z)*z)*z)*(((y*y)*y)*(((x*(x*z))*y)*((((((((y*(z*z))*(z*y))*(y*(x*y)))*(z*(y*(z*x))))*(((x*(y*(y*y)))*z)*z))*(x*((z*y)*x)))*((z*z)*(z*x)))))))))*(z*(y*x)))))))))*(y*z))*(((y*x)*x)*z)))*((x*x)*(x*y)))*((y*(x*z))*z))*(y*(x*y)))*(x*((y*z)*y)))*(x*y))*(z*(x*(z*z))))*(((y*z)*y)*y)))*(y*((z*x)*(x*z))))))*(((x*(x*x))*((((z*(y*y))*((((z*y)*(x*x))*((((((((x*(y*y))*z)*(((x*(x*(y*z)))*((((x*x)*(z*z))*((((z*y)*(x*y))*(((x*((z*y)*y))*(((y*x)*(z*x))*(((y*(z*y))*z)*(((y*((((((y*(z*z))*z)*((y*y)*(((z*x)*(x*(z*(x*y))))))*((x*(x*y))*z))*(z*((z*z)*z))))*((z*((z*y)*x))*z))*(x*((y*z)*x))))))*(y*((z*z)*z)))*((y*(x*x))*x)))*((y*(x*z))*x))))*(((z*z)*z)*y))*(((y*z)*y)*(x*y))))*((x*x)*z))*(((z*x)*y)*z))*(y*(y*(x*z))))*((z*z)*z))*((x*(z*z))*y)))*(z*((z*x)*y)))*((z*x)*(z*z))))*(x*((x*y)*z)))*((z*(x*z))*y)))*(((z*z)*$

$$(((z*(y*x))*((z*y)*(((y*((x*x)*x))*((z*z)*(((((z*z)*(x*z))*((x*(z*z))*((x*(y*(z*y)))*(((y*y)*x)*(((x*x)*(((y*z)*(z*x))*((x*z)*(x*(((x*x)*(z*x))*(x*((x*z)*y)))*(x*(z*y))))))))*(y*((x*z)*y))))*((y*y)*(x*x)))))*((y*y)*x))*((y*x)*(y*z)))))*(x*((x*x)*y)))))*((z*y)*x)))*(((x*((x*x)*x))*(x*(((y*x)*z)*(((z*y)*y)*((((x*(((z*(y*(x*y)))*(((z*(z*z))*y)*(((((z*(y*(z*x)))*(((x*y)*(z*x))*((z*(y*y))*((x*(x*y))*(((x*y)*y)*((z*y)*(((x*z)*x)*(((x*x)*((z*x)*y))*((x*y)*y)))))))))*(((z*y)*z)*z))*(x*(y*(z*z))))*x)*(((y*z)*y)*z))))*(z*(x*x))))*(y*(z*x)))*y)*x)))))*(((x*(x*x))*((x*(y*(y*z)))*(((((z*y)*(x*x))*((x*(y*y))*((x*((y*x)*x))*(((((z*y)*(z*y))*((x*(x*(((y*(y*z))*z)*z))*((z*((z*z)*z))*((((z*(z*y))*z)*z)*(((((x*y)*y)*(((x*z)*(z*z))*x))*(((x*x)*y)*x)))))))*(x*((y*x)*z))))*(z*((x*y)*y)))*(x*(z*(y*y))))))*(((z*x)*y)*z))*((z*(y*z))*y))))*((((z*x)*(((y*x)*y)*x))*((y*x)*(((x*x)*y)*(((((x*y)*z)*z)*((z*((y*y)*z))*((y*(z*x))*(((z*y)*y)*((y*x)*(((x*(z*z))*x)*(((z*y)*y)*((y*(x*y))*((((x*(x*(x*z)))*((z*((z*y)*x))*((y*(y*z))*$$

$$(((z*y)*(y*x))*((y*((y*z)*y))*((((x*z)*z)*x)*(x*y))))))*(z*y))*(x*(y*(x*x)))))))))))))*((((x*(y*(x*z)))*(((y*(y*x))*y)*((y*((y*z)*z))*((y*(x*(z*z)))*((x*z)*(((z*z)*((((y*y)*(x*x))*y)*(((x*(z*z))*z)*(z*(z*(x*z)))*(x*z)))))*(x*(y*y)))))))))*((((((((((x*(z*z))*z)*(((x*(z*y))*z)*((z*(y*((y*y)*y)))*((z*((y*(z*x))*x))*((z*x)*(((y*x)*(z*y))*(((y*y)*(y*z))*((y*(z*y))*(x*y))))))))*(((x*(y*(y*y)))*((x*(y*x))*((y*(y*(x*x)))*((((z*z)*z)*y)*(z*((z*((z*y)*z))*(y*y))))))))*((((((((x*z)*(y*z))*((z*(y*(y*x)))*x))*(((x*(z*(z*x)))*(y*y))*(((((y*y)*z)*z)*(y*z))*((((z*x)*y)*z)*(((y*(y*(y*x)))*z)*((((z*z)*z)*z)*((((y*((y*x)*x))*z)*x)*((x*x)*y))))))*(((((y*y)*z)*z)*x)))*((y*(x*(z*z)))*y))))*((y*(x*(x*z)))*(y*x))))*((y*(z*(y*z)))*x))))*(((((y*y)*y)*y)*y)*y))*(((y*z)*(x*(y*z)))*((y*x)*(((y*(z*x))*x)*((x*(z*z))*(((z*x)*x)*(x*x))))))*((z*(x*(z*y)))*x))*((x*z)*y))))*((z*z)*(x*x)))*((z*((y*z)*x))*((x*(z*(x*(y*z))))*((y*z)*(x*y)))))*(((y*(y*y))*y)*(x*z)))*(((z*y)*y)*((y*(y*x))*$$

$$(((x*z)*y)*(((((x*(z*z))*z)*((x*x)*(((y*y)*(y*z))*(z*y))))*(z*((x*z)*y)))*((x*(y*z))*y)))))*(((x*x)*y)*((x*((x*x)*z))*((z*(y*(z*x)))*z)))))*((((x*z)*z)*x)*(((z*x)*(z*y))*((x*(x*(y*z)))*(((x*(z*x))*(((x*(y*(x*(z*x))))*((y*x)*((((z*x)*z)*z)*(((y*y)*((y*y)*(z*z)))*((x*x)*z)))))*(x*(x*x))))*((z*x)*x)))))))*(((x*z)*(x*x))*((((z*y)*x)*((z*(z*(y*z)))*(((((x*x)*z)*x)*(((((((z*x)*y)*((y*z)*((y*y)*x)))*((x*(x*x))*z)))*(y*(x*y)))*(z*((y*z)*y)))*((y*(y*z))*y))*(x*z))*x))*(((y*(y*x))*z)*z))*(z*(y*(x*y))))))*(x*((y*z)*y))))))))$$

Thanks for listening!

— DC